University School of Automation and Robotics
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USAR
UNIVERSITY SCHOOL OF AUTOMATION AND ROBOTICS

Engineering Mechanics

By: Dr. Divya Agarwal

- **UNIT- I**

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon's theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- **Distributed forces:** Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

- **UNIT- II**

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.

- **UNIT-III**

- **Kinematics of Particles:** Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates

- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

- **UNIT-IV**

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.

- **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.

- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple

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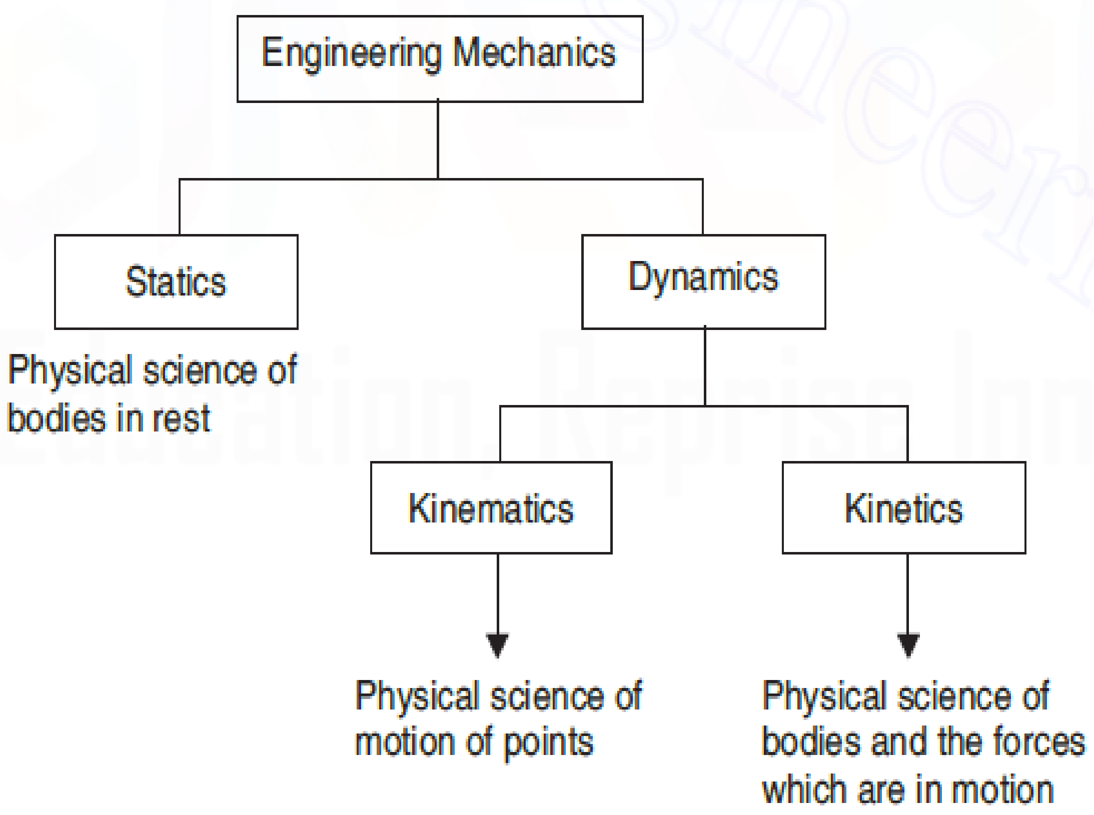
INTRODUCTION TO MECHANICS

- Physical science which deals with the state of rest or motion of bodies under the action of force
- It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces. Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.
- **Why we study mechanics?**
  - This science form the groundwork for further study in the design and analysis of structures
- Depending upon the nature of the body involved, it can be further divided into mechanics of rigid body, mechanics of deformable bodies (also called strength of materials) and mechanics of fluids.
- Rigid bodies are those which do not deform under the action of applied forces. The mechanics of rigid bodies is studied in two parts: - statics and dynamics.



BASIC TERMINOLOGY

- **Statics:** Statics deal with the condition of equilibrium of a rigid-body at rest
- **Rigid body:** A rigid body is defined as a definite quantity of matter, the parts of which are fixed in position relative to each other. Physical bodies are never absolute but deform slightly under the action of loads. If the deformation is negligible as compared to its size, the body is termed as rigid. A rigid body is one which does not change its shape and size under the effect of force acting over it. It differs from an elastic body in the sense that the later undergoes deformation under the effect of forces acting on it and return to its original shape and size on removal of the forces acting on the body. The rigidity of a body depends upon the fact that how far it undergoes deformation under the effect of forces acting on it.
- **Dynamics:** dealing with a rigid-body in motion
- **Length:** This term is applied to the linear dimensions of a straight or curved line. For example, the diameter of circle is the length of a straight line which divides the circle into two equal parts ; the circumference is the length of its curved perimeter.
- **Area:** The two dimensional size of a shape or a surface is its area. The shape may be flat (lie in a plane) or curved, for example, the size of a plot of land, the surface of a fluorescent bulb, or the cross-sectional size of a shaft.

BASIC TERMINOLOGY

- **Volume:** The three dimensional or cubic measure of the space occupied by a substance is known as its volume.
- **Force:** This term is applied to any action on the body which tends to make it move, change its motion, or change its size and shape. A force is usually thought of a push or pull, such as a hand pushing against a wall or the pull of a rope fastened to a body.
- **Pressure:** The external force per unit area, or the total force divided by the total area on which it acts, is known as pressure. Water pressure against the face of a dam, steam pressure in a boiler, or earth pressure against a retaining wall are some examples.
- **Mass:** The amount of matter contained in a body is called its mass, and for most problems in mechanics, mass may be considered constant.
- **Weight:** The force with which a body is attracted towards the centre of earth by the gravitational pull is called its weight.
- **Particle:** a body of negligible dimension. It is defined as an object whose mass is concentrated at a point. This assumption is made when the size of a body is negligible and is irrelevant to the description of the motion of the body.

BASIC CONCEPTS

- The study of mechanics involves the concepts of space, time, mass and force.
  i. Concept of space is essential to fix the position of a point. To fully define the position of a point in space we shall need to define some frame of reference and coordinate system.
  ii. Concept of time is essential to relate the sequence of events, for example, starting and stopping of the motion of a body.
  iii. Concept of mass is essential to distinguish between the behaviour of two bodies under the action of an identical force.
  iv. Concept of force is essential as an agency which changes or tends to change the state of rest or of uniform motion of body.
- To describe the state of rest or motion of a body, some reference is required. A body is said to be at rest or in motion only with respect to some reference frame. This reference preferably should be fixed in space. As it is doubtful to locate any fixed reference in the universe, so the earth surface is usually employed as a reference frame. Such a reference serves as an inertial frame. A truly inertial frame is one which moves at constant velocity.

QUESTIONS: FILL IN THE BLANKS

(*i*) ......is the branch of mechanics which relates to bodies at rest.

(*ii*) ......is the branch of mechanics which deals with bodies in motion.

(*iii*) ......deals with the motion of the bodies without any reference to the cause of motion.

(*iv*) ......deals with relationship between forces and the resulting motion of bodies on which they act.

(*v*) ......is the term applied to the linear dimensions of a straight or curved line.

(*vi*) External force per unit area, or total force divided by the total area on which it acts, is known as......

(*vii*) The amount of matter contained in a body is called its......

(*viii*) ......= torque × angle.

(*ix*) The rate of doing work is called......

(*x*) A scalar quantity is one that has......only.

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Answers: (*i*) Statics (*ii*) Dynamics (*iii*) Kinematics (*iv*) Kinetics (*v*) Length (*vi*) pressure (*vii*) mass (*viii*) Work (*ix*) power (*x*) magnitude.

QUESTIONS: SAY 'YES' OR 'NO'

(*i*) The two dimensional size of a shape or a surface is its area.

(*ii*) Mass helps motion.

(*iii*) Velocity is a vector quantity.

(*iv*) A rigid body is one which undergoes change in its shape and size under effect of forces acting over it.

(*v*) Acceleration is a scalar quantity.

(*vi*) In real sense, no solid body is perfectly rigid.

(*vii*) There is no difference between mass and weight.

(*viii*) The tendency of a force to cause rotation about some point is known as a moment.

(*ix*) Mass can be measured by a spring balance.

(*x*) Weight resists motion in the body.

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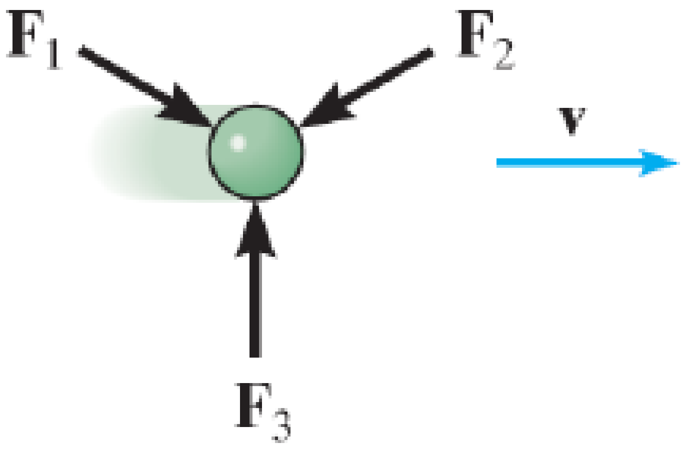
FUNDAMENTAL PRINCIPLES

The elementary mechanics rests on a few fundamental principles based on experimental observations. These are:

1. Newton's Three laws of motion
2. Newton's Law of Gravitation
3. The Principle of Transmissibility of Force
4. The Parallelogram Law for the Addition of Forces

NEWTON'S FIRST LAW OF MOTION

- A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.
- First law contains the principle of the equilibrium of forces which is the main topic of concern in Statics
- As per first law, everybody continues in a state of rest or of uniform motion in a straight line unless it is compelled to change that state by a force imposed on the body. Thus first law of motion helps us to define a force



Equilibrium

NEWTON'S SECOND LAW OF MOTION

- A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force.
- Second Law forms the basis for most of the analysis in Dynamics

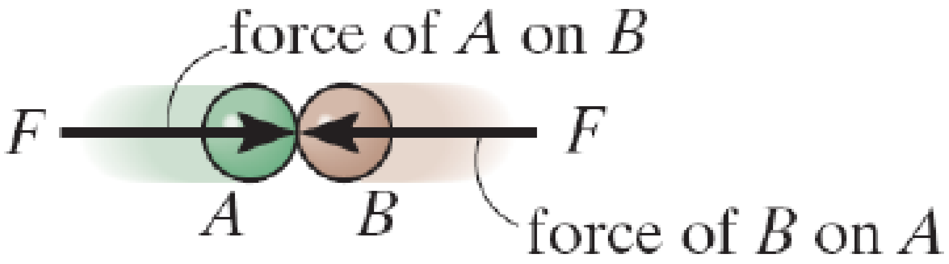
Accelerated motion

$$F = ma$$

- The acceleration of a given particle is proportional to the impressed force and takes place in the direction of the straight line in which the force is impressed. This law helps us to measure force quantatively.

NEWTON'S THIRD LAW OF MOTION

- The mutual forces of action and reaction between two particles are equal, opposite, and collinear.
- Third law is basic to our understanding of Force that forces always occur in pairs of equal and opposite forces



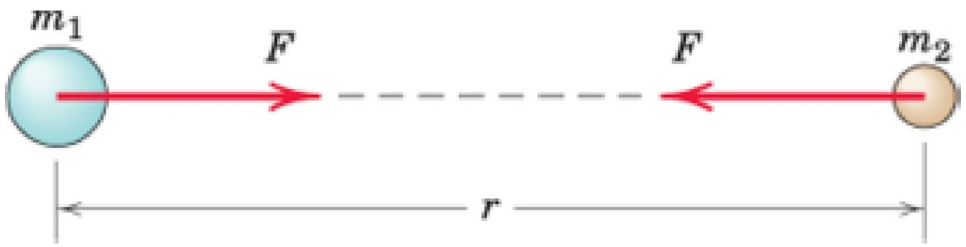
Action – reaction

- To every action there is equal and opposite reaction. Which means, that the forces of action and reaction between two bodies are equal in magnitude but opposite in direction.

NEWTON'S LAW OF GRAVITATION

- Weight of a body (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics. This law governs the gravitational attraction between any two particles.
- Weight of a Body: If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle
- Two particles are attracted towards each other along with line connecting them with a force whose magnitude is proportional to the product of their masses and inversely proportional to the square of distance between them.

$$F = G\frac{m_1 m_2}{r^2}$$



- ***F*** = mutual force of attraction between two particles
- ***G*** = universal constant of gravitation
- [For experiments take $G = 6.673 \times 10^{-11}$ $m^3/(kg.s^2)$]
- **Rotation of Earth is not taken into account**
- $\boldsymbol{m_1}$, $\boldsymbol{m_2}$ = masses of two particles
- ***r*** = distance between two particles

NEWTON'S LAW OF GRAVITATION

- The force of attraction exerted by the earth on a particle lying on its surface is governed by this law.
- If a particle of mass m lies on the surface of the earth of mass M_e and radius r (***r*** = distance between the earth's center and the particle), the force exerted by the earth if equal to weight W of the particle.
- Therefore

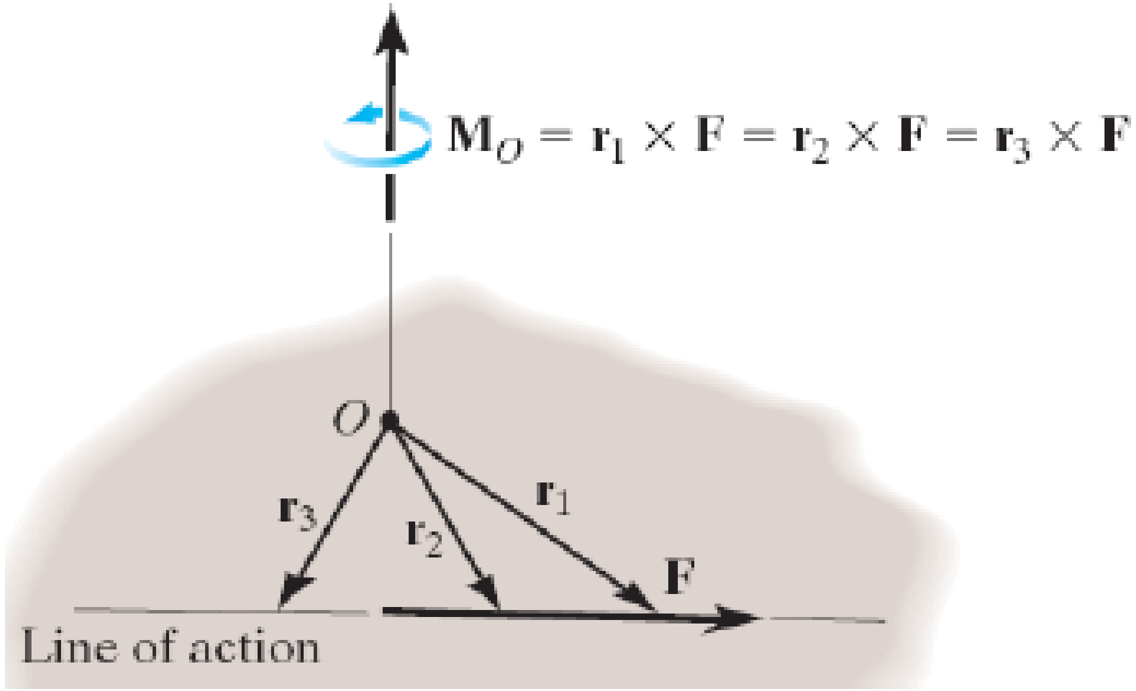
$$W = G\frac{mM_e}{r^2}$$

$$W = mg$$

- Let $\boldsymbol{g} = \boldsymbol{G\,M_e\,/r^2}$ = acceleration due to gravity ($9.81 m/s^2$)

PRINCIPLE OF TRANSMISSIBILITY OF A FORCE

- It states that the condition of equilibrium or of the motion of the rigid body will remain unchanged if the point of application of a force acting on the rigid body is transmitted to act at any other point along its line of action.
- The principle of transmissibility of forces states that *when a force acts upon a body, its effect is the same whatever point in its line of action is taken as the point of the application provided that the point is connected with the rest of the body in the same invariable manner.*



PRINCIPLE OF TRANSMISSIBILITY OF A FORCE

- A force may be considered as acting at any point on its line of action so long as the direction and magnitude are not changed.
- Suppose a body (Fig. 1) is to be moved by a horizontal force *P* applied by hooking a rope to some point on the body. The force *P* will have the same effect if it is applied at 1, 2, 3 (Fig. 2) or any point on its line of action. This property of force is called *transmissibility*.

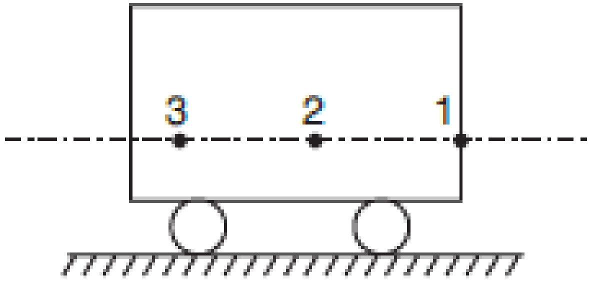


Fig 1

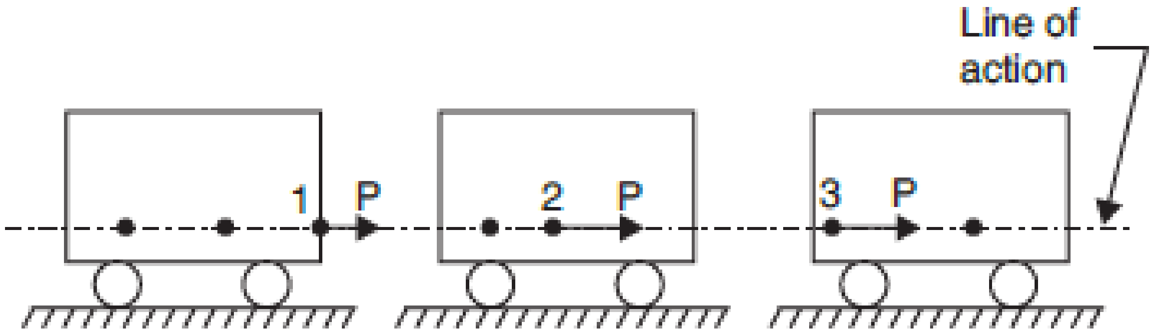


Fig 2

PRINCIPLE OF TRANSMISSIBILITY OF A FORCE

- A force F acting on the rigid body at point B, having the line of action ab, can be replaced by the same force F but acting at the point A provided this new point A lies along with line of action ab of the force. (See figure)
- In other words, the force F acting at point A can be transmitted to act at any other point B along its line of action without changing its effect on the rigid body.

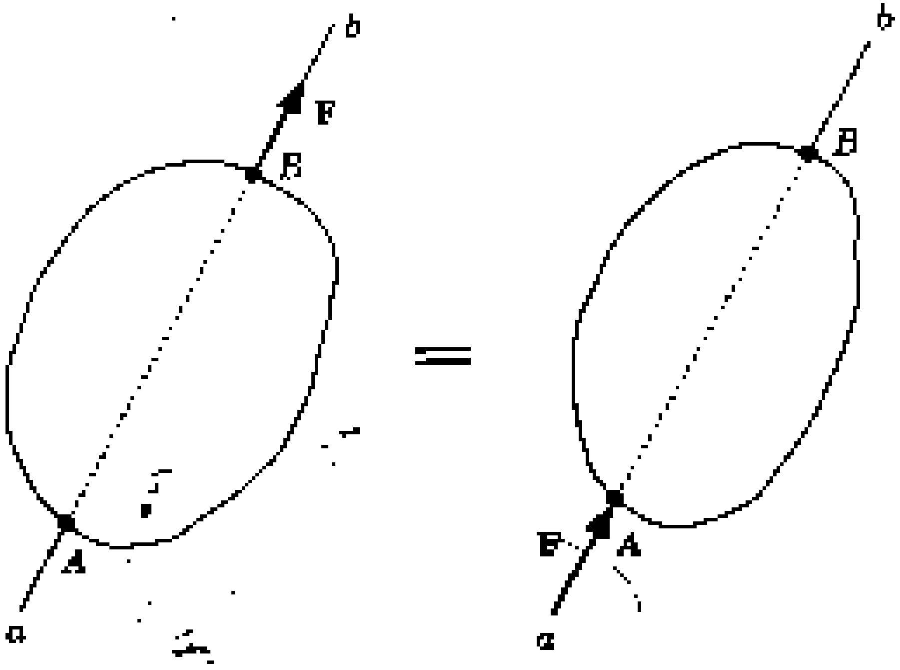
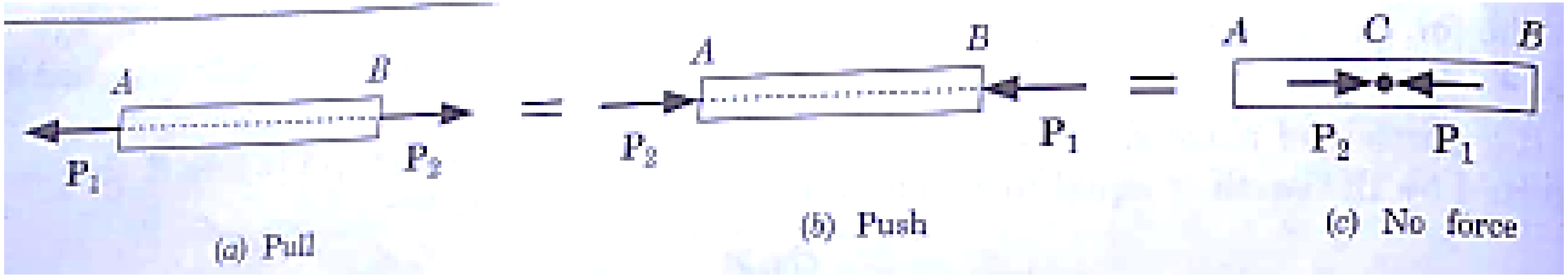


Fig 1

PRINCIPLE OF TRANSMISSIBILITY OF A FORCE

- Next consider a prismatic bar AB which is acted upon by two equal and opposite coaxial forces P_1 and P_2.

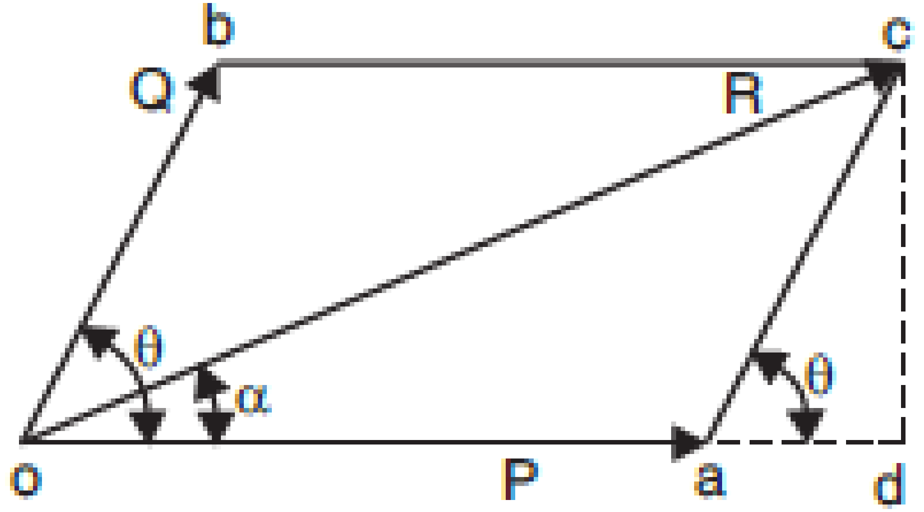


- Using the principle of transmissibility, the force P_1 acting at point A can be transmitted to act at point B and the force P_2 acting at point B can be transmitted to act at A. This new system of forces acting on the bar, as obtained by the principle of transmissibility, does not change the condition of equilibrium of the body.
- But from the point of view of internal forces, the state of tension (pull) has been changed to a state of compression (push). i.e., tendency of elongating the bar has been changed to that of shorting the bar.
- Thus, this principle can be used to discuss the condition of equilibrium or motion of rigid body and to determine the external forces acting on the rigid body. But it should not be used to determine the internal forces and deformations of the body.

THE PARALLELOGRAM LAW

- If two forces acting at a point are represented in magnitude and direction by the adjacent sides of a parallelogram then the diagonal of the parallelogram passing through their point of intersection represents the resultant in both magnitude and direction. This implies that a force can be represented by a straight line with an arrow in the manner of a vector.

- Refer Fig. Let two forces *P* and *Q* acting simultaneously on a particle be represented in magnitude and direction by the adjacent sides *oa* and *ob* of a parallelogram *oacb* drawn from a point *o*, their resultant *R* will be represented in magnitude and direction by the diagonal *oc* of the parallelogram.



- The value of *R* can be determined either graphically or analytically as explained below :

FORCES IN A PLANE

- Force is some thing which changes or tends to change the state of rest or of uniform motion of a body in a straight line.
- Force is the direct or indirect action of one body on another.
- The bodies may be in direct contact with each other causing direct motion or separated by distance but subjected to gravitational effects.
- There are different kinds of forces such as gravitational, frictional, magnetic, inertia or those cause by mass and acceleration.
- A static force is the one which is caused without relative acceleration of the bodies in question.
- The force has a magnitude and direction, therefore, it is vector.

FORCES IN A PLANE

- While the directions of the force is measured in absolute terms of angle relative to a co-ordinate system, the magnitude is measured in different units depending on the situation.
- When a force acts on a body, the following effects may be produced in that body :

(i) It may bring a change in the motion of the body i.e., the motion may be accelerated or retarded ;

(ii) it may balance the forces already acting on the body thus bringing the body to a state of rest or of equilibrium, and

(iii) it may change the size or shape of the body i.e., the body may be twisted, bent, stretched, compressed or otherwise distorted by the action of the force.

CHARACTERISTICS OF A FORCE

- The characteristics or elements of the force are the quantities by which a force is fully represented. These are :

1. Magnitude (*i.e.*, 50 N, 100 N, etc.)
2. Direction or line of action (angle relative to a co-ordinate system).
3. Sense or nature (push or pull).
4. Point of application.

SCALARS AND VECTORS

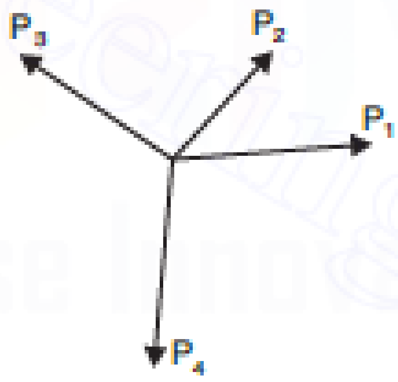
- We are familiar with the quantities like mass, time, volume and energy which can be completely defined by stating their magnitude and which do not have any direction.
- They can be added and subtracted according to the law of algebra. Such quantities are called scalar quantities.
- On the other hand, quantities like displacement, velocity, acceleration, momentum and force possess both; magnitude as well as direction. To define these quantities we have to specify their magnitude, direction and point of action. Such quantities can be added according to the parallelogram law, and are termed as vector quantities.
- A vector is represented as $\overrightarrow{P}$

FORCE SYSTEMS

- A *force system* is a collection of forces acting on a body in one or more planes.
- According to the relative positions of the lines of action of the forces, the forces may be classified as follows :

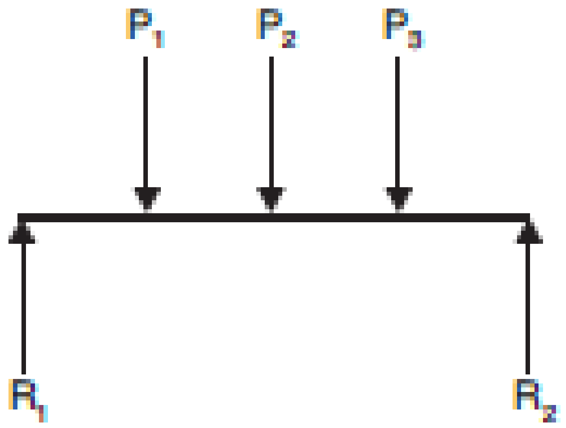
1. **Coplanar concurrent collinear force system.** It is the simplest force system and includes those forces whose vectors lie along the same straight line (refer Fig. 1.).

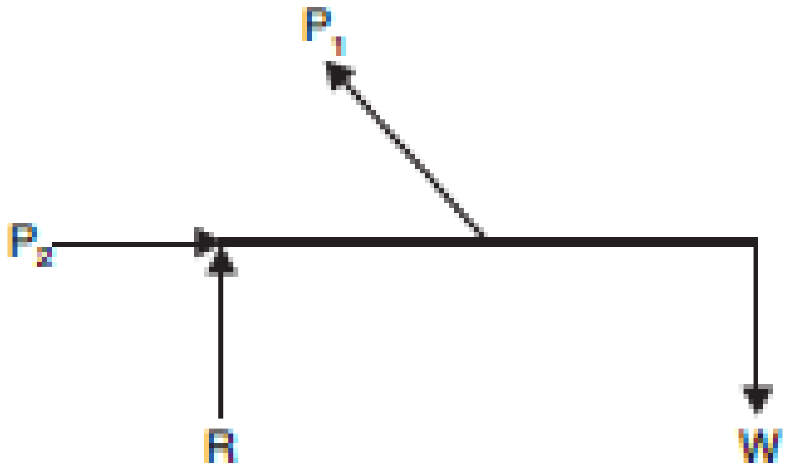
2. **Coplanar concurrent non-parallel force system.** Forces whose lines of action pass through a common point are called **concurrent forces.** In this system lines of action of all the forces meet at a point but have different directions in the same plane as shown in Fig. 2



FORCE SYSTEMS

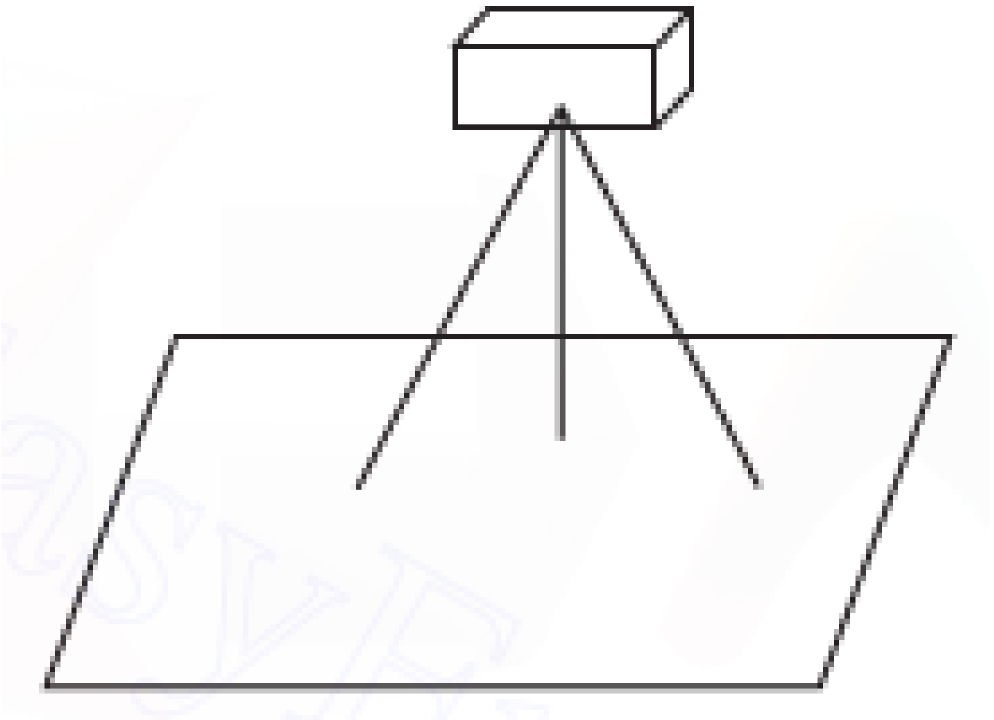
3. **Coplanar non-concurrent parallel force system.** In this system, the lines of action of all the forces lie in the same plane and are parallel to each other but may not have same direction as shown in Fig. 3.
4. **Coplanar non-concurrent non-parallel force system.** Such a system exists where the lines of action of all forces lie in the same plane but do not pass through a common point. Fig. 4 shows such a force system.



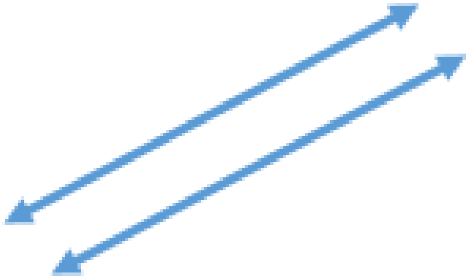


FORCE SYSTEMS

5. **Non-coplanar concurrent force system.** This system is evident where the lines of action of all forces do not lie in the same plane but do pass through a common point. An example of this force system is the forces in the legs of tripod support for camera (Fig. 5).
6. **Non-coplanar non-concurrent force system.** Where the lines of action of all forces do not lie in the same plane and do not pass through a common point, a non-coplanar non-concurrent system is present.

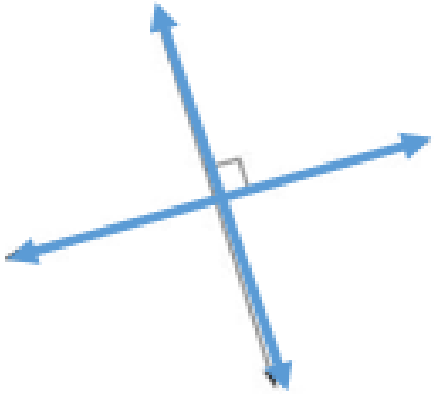


EXAMPLE OF TYPE OF LINES

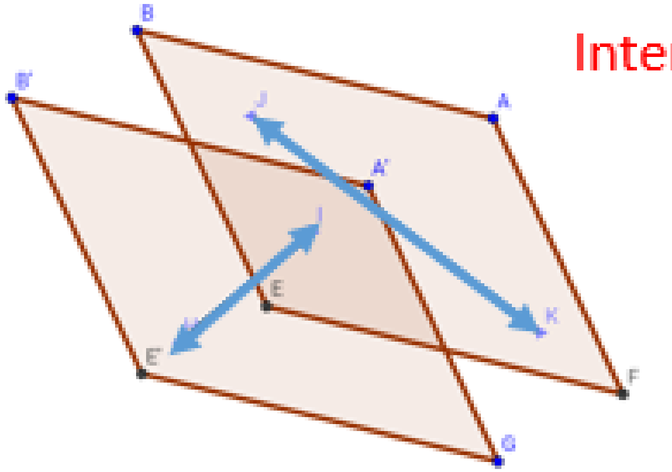


Parallel lines stay the same distance apart and never cross.

Intersecting lines cross at one point.



Perpendicular lines intersect and form right angles.



Skew lines are on different planes and do not intersect.

RESULTANT FORCE

- A *resultant force is a single force which can replace two or more forces and produce the same effect on the body as the forces.*
- When a number of forces acting on a rigid body are replaced by a single force which has the same effect on the rigid body as that of all the forces acting together then this single force is called the resultant of several forces.
- It is fundamental principle of mechanics, demonstrated by experiment, that when a force acts on a body which is free to move, the motion of the body is in the direction of the force, and the distance travelled in a unit time depends on the magnitude of the force.
- Then, for a system of concurrent forces acting on a body, the body will move in the direction of the resultant of that system, and the distance travelled in a unit time will depend on the magnitude of the *resultant.*

COMPONENT OF A FORCE

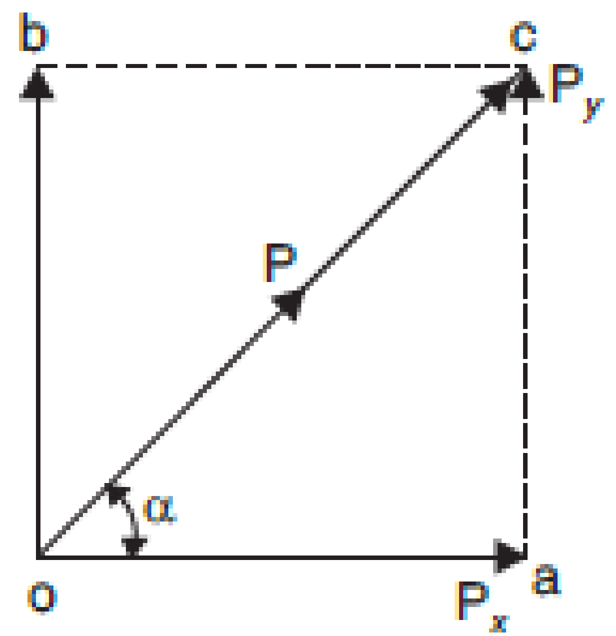
- As two forces acting simultaneously on a particle acting along directions inclined to each other can be replaced by a single force which produces the same effect as the given force.
- Similarly, a single force can be replaced by two forces acting in directions which will produce the same effect as the given force. This breaking up of a force into two parts is called the *resolution of a force.*
- The force which is broken into two parts is called the *resolved force* and the parts are called *component forces* or the *resolute.*
- Generally, a force is resolved into the following two types of components :
- 1. Mutually perpendicular components
- 2. Non-perpendicular components.

COMPONENT OF A FORCE - MUTUALLY PERPENDICULAR COMPONENTS

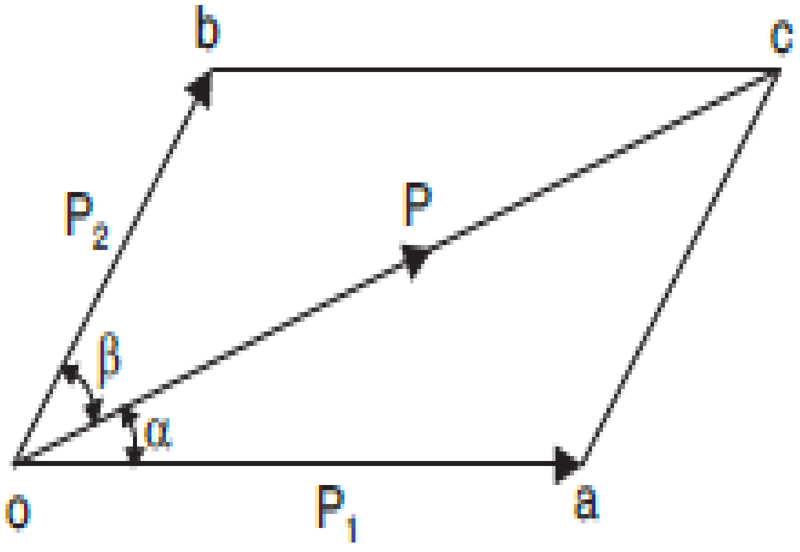
- Let the force P to be resolved is represented in magnitude and direction by *oc* in Fig.
- Let P_x is the component of force P in the direction *oa* making an angle α with the direction *oc* of the force.
- Complete the rectangle *oacb*.
- Then the other component P_y at right angle to P_x will be represented by *ob* which is also equal to *ac*.
- From the right-angled triangle *oac*

$$P_x = oa = P \cos \alpha$$

$$P_y = ac = P \sin \alpha.$$



COMPONENT OF A FORCE- NON-PERPENDICULAR COMPONENTS



Now from the triangle *oac*, by applying sine rule,

$$\frac{oa}{\sin\beta} = \frac{oc}{\sin[180 - (\alpha + \beta)]} = \frac{ac}{\sin\alpha}$$

or

$$\frac{P_1}{\sin\beta} = \frac{P}{\sin(\alpha + \beta)} = \frac{P_2}{\sin\alpha}$$

∴

$$P_1 = P \cdot \frac{\sin\beta}{\sin(\alpha + \beta)}$$

and

$$P_2 = P \cdot \frac{\sin\alpha}{\sin(\alpha + \beta)}$$

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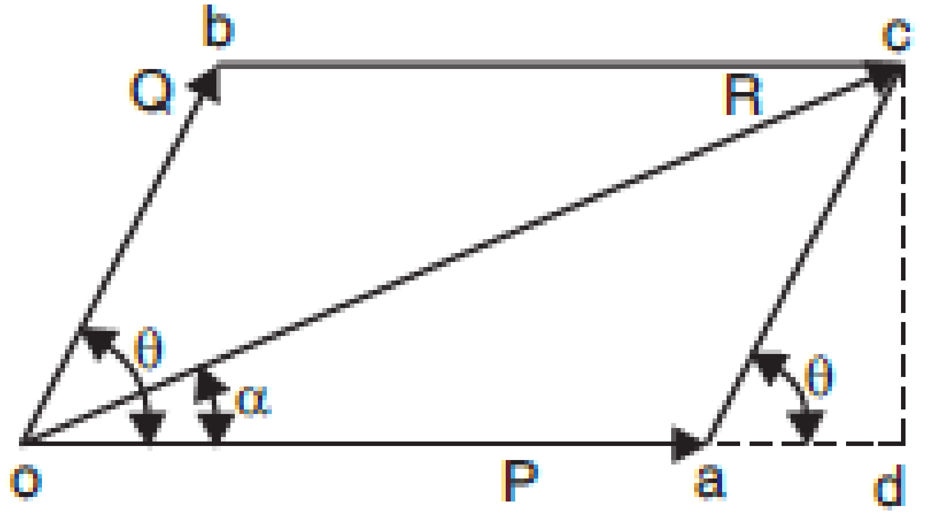
- If two forces acting at a point are represented in magnitude and direction by the adjacent sides of a parallelogram then the diagonal of the parallelogram passing through their point of intersection represents the resultant in both magnitude and direction. This implies that a force can be represented by a straight line with an arrow in the manner of a vector.

- Refer Fig. Let two forces P and Q acting simultaneously on a particle be represented in magnitude and direction by the adjacent sides oa and ob of a parallelogram oacb drawn from a point o, their resultant R will be represented in magnitude and direction by the diagonal oc of the parallelogram.



- The value of R can be determined either graphically or analytically as explained below :

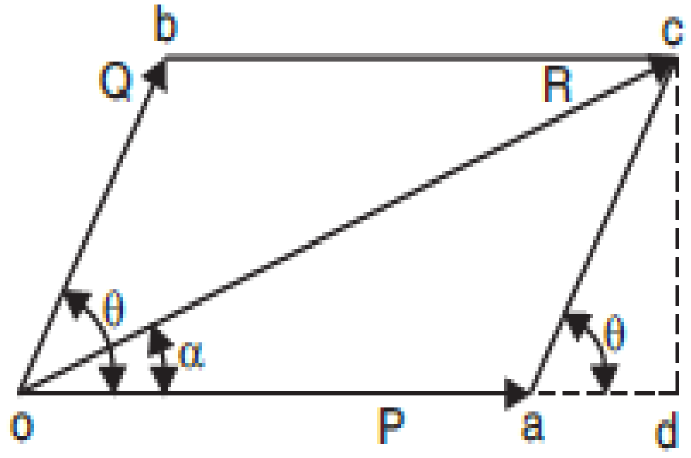
THE PARALLELOGRAM LAW

- **Graphical method.** Draw vectors *oa* and *ob* to represent to some convenient scale the forces *P* and *Q* in magnitude and direction.
- Complete the parallelogram *oacb* by drawing *ac* parallel to *ob* and *bc* parallel to *oa*. The *vector oc* measured to the same scale will represent the resultant force *R*.
- **Analytical method.** As shown in Fig. , in the parallelogram *oacb*, from *c* drop a perpendicular *cd* to *oa* at *d* when produced.
- Now from the geometry of the figure.

$\angle cad = \theta$, $ac = Q$

$\therefore cd = Q \sin \theta$

and $ad = Q \cos \theta$



THE PARALLELOGRAM LAW

From right-angled triangle, *odc*

$$oc = \sqrt{(od)^2 + (cd)^2}$$

$$= \sqrt{(oa + ad)^2 + (cd)^2}$$

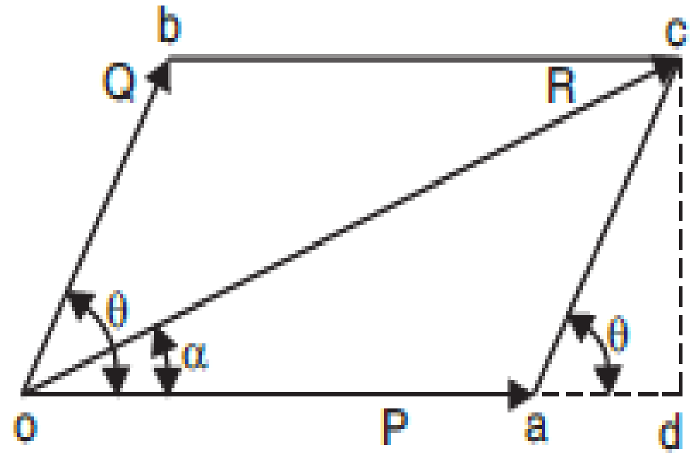
or $$R = \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2}$$

$$= \sqrt{P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta}$$

$$= \sqrt{P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta}$$

$$= \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

$\therefore$ $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$



THE PARALLELOGRAM LAW

Let the resultant makes an angle α with P as shown in figure.

Then $$\tan \alpha = \frac{cd}{od} = \frac{cd}{oa + ad}$$

$$= \frac{Q \sin \theta}{P + Q \cos \theta}$$

Case 1. If $\theta = 0°$, *i.e.*, when the forces P and Q act along the same straight line then equation (2.3) reduces to

$$R = P + Q \qquad (\because \cos 0° = 1)$$

Case 2. If $\theta = 90°$, *i.e.*, when the forces P and Q act at right angles to each other, then

$$R = \sqrt{P^2 + Q^2} \qquad (\because \cos 90° = 0)$$

Case 3. If $\theta = 180°$, *i.e.*, the forces P and Q act along the same straight line but in opposite directions, then

$$R = P - Q \qquad (\because \cos 180° = -1)$$

The resultant will act in the direction of the greater force.

TRIANGLE LAW OF FORCES.

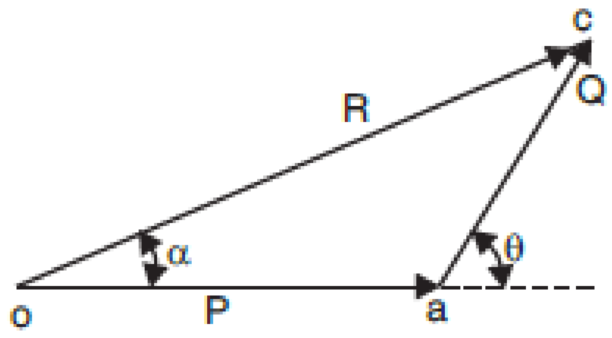
- It states as under :*"If two forces acting simultaneously on a body are represented in magnitude and direction by the two sides of triangle taken in order then their resultant may be represented in magnitude and direction by the third side taken in opposite order."*
- Let P and Q be the two coplanar concurrent forces.
- Resultant force R in this case can be obtained with the help of the triangle law of forces both graphically and analytically as given below :
- **Graphical method.** Refer Fig. Draw vectors *oa* and *ac* to represent the forces P and Q to some convenient scale in magnitude and direction. Join *oc* which will represent the resultant force R in magnitude and direction to the same scale.
- **Analytical method.** From the geometry of triangle *oac* (Fig. shown below).

$$\angle coa = \alpha,\ \angle oca = \theta - \alpha,\ \angle cao = 180° - \theta$$

$$\therefore \quad \frac{oa}{\sin(\theta - \alpha)} = \frac{ac}{\sin \alpha} = \frac{oc}{\sin(180° - \theta)}$$

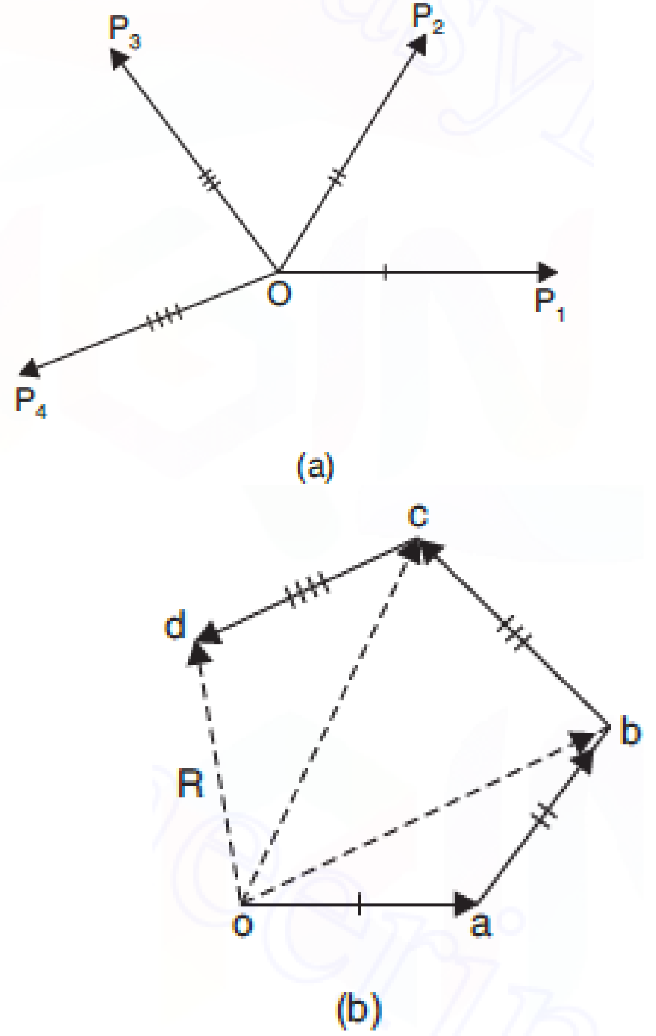
$$\text{or} \quad \frac{P}{\sin(\theta - \alpha)} = \frac{Q}{\sin \alpha} = \frac{R}{\sin(180° - \theta)}$$

$$\text{or} \quad \frac{P}{\sin(\theta - \alpha)} = \frac{Q}{\sin \alpha} = \frac{R}{\sin \theta}$$



POLYGON LAW OF FORCES

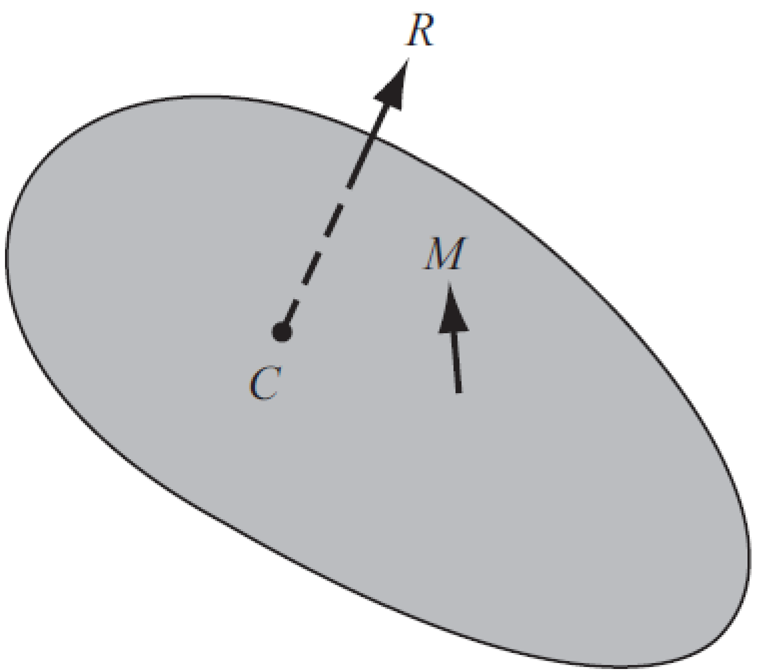
- It states as under :"*If a number of coplanar concurrent forces, acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order, then their resultant may be represented in magnitude and direction by the closing side of a polygon, taken in the opposite order*"
- If the forces P_1, P_2, P_3, and P_4 acting simultaneously on a particle be represented in magnitude and direction by the sides *oa, ab, bc* and *cd* of a polygon respectively, their resultant is represented by the closing side *do* in the opposite direction as shown in Fig. (*b*).
- The law is actually an extension of triangle law of forces. This is so because *ob* is the resultant of *oa* and *ab* and therefore *oc* which is resultant of *ob* and *bc* is also the resultant of *oa, ab* and *bc*.
- Similarly, *od* is the resultant of *oc* and *cd* and therefore of *ob, bc* and *cd* and finally of *oa, ab, bc* and *cd*.



RESULTANT OF A FORCE SYSTEM

- Resultant action on a body implies the net external action on the body.
- In other words, the resultant action is a simple equivalent force system which can replace the given force system for an equivalence of effect so far as motion or tendency of motion of the body is concerned.
- The definition of a rigid body permits no internal dimensional or structural changes within the body; the resultant concept applied to a rigid body stands for complete equivalence of action and is therefore highly meaningful.
- Also, a particle conceived as a relatively small object allows resultant concept to be used to advantage due to complete equivalence of action represented by it.
- We, confine ourselves to the resultant concept for a particle and a rigid body.
- Action of a force is two-fold: first, in its own right as a translational action and second, to generate a moment or rotational action about an arbitrary point / axis.
- It follows that the resultant of a force system should, in general, comprise of

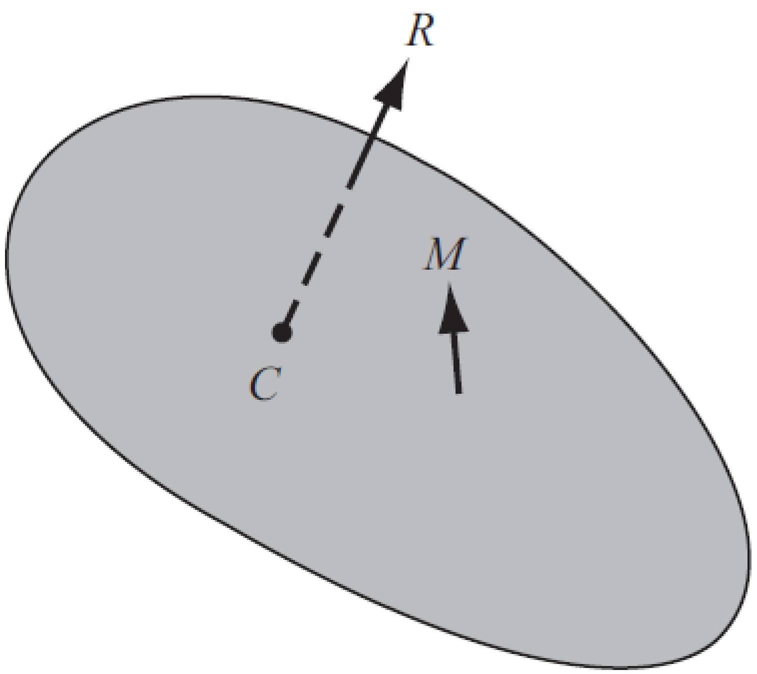
  (a) a force and (b) a moment, as shown in Figure.



Resultant action for the force system acting on a body

RESULTANT OF A FORCE SYSTEM

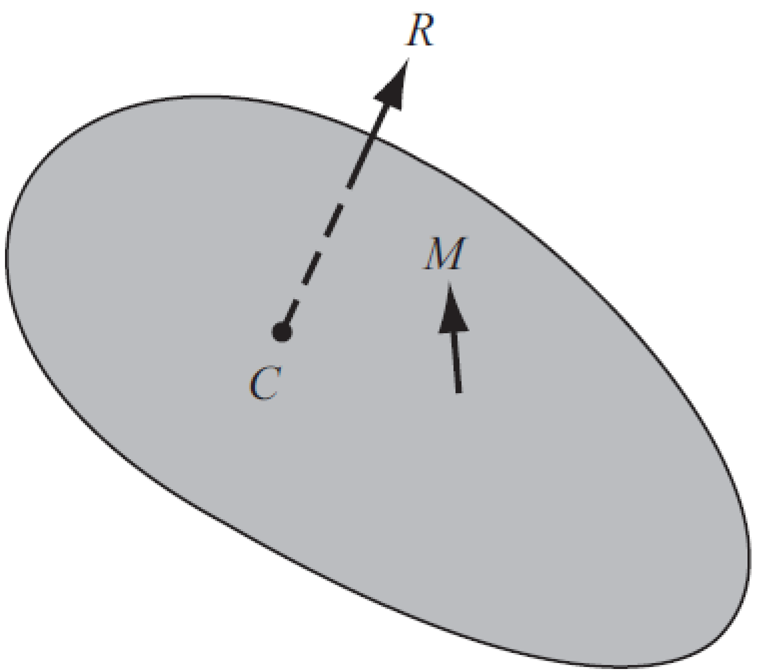
- It is necessary to qualify the point of action or line of action of the force and the direction of the moment as will be shown later.
- It is also likely that a force system can result in a force only or in a moment only for certain force fields and certain choices of the point of action of the resultant.
- Sometimes, the resultant of a force system is referred to as the equivalent or equipollent action.
- It is, in general, incorrect to term the resultant as the equivalent because the equivalence of an action has a wider implication.
- The equivalence of action on a body may be desired with different objectives.
- For example, the objective may be to study the motion of the body, to analyse its internal forces or to compute its deformation or rate of deformation.
- Thus, by definition, the equivalent system of forces for a given system of forces is such that it produces the same desired effect.



Resultant action for the force system acting on a body

RESULTANT OF A FORCE SYSTEM

- In particular, for a rigid body in motion or having a tendency of motion, the analysis of motion can be made with the resultant replacing the given system of forces.
- The resultant is, therefore, the equivalent action of a system of forces for the dynamic consideration of a rigid body.
- It may be seen that the resultant of a plane system of forces must be a force in that plane which may be accompanied by a couple-moment in a direction normal to that plane.
- Similarly, the resultant of a system of parallel forces should be a force parallel to them which may be accompanied by a moment in a direction normal to the parallel forces.
- The resultant of a system of concurrent forces should be a single force which must pass through the point of concurrency.
- Let us discuss the individual cases.



Resultant action for the force system acting on a body

RESULTANT OF A CONCURRENT FORCE SYSTEM

- A concurrent force system may be collinear, coplanar or spatial;

I. Collinear if the forces have the same line of action,

II. Coplanar if the lines of action of the forces lie in a plane

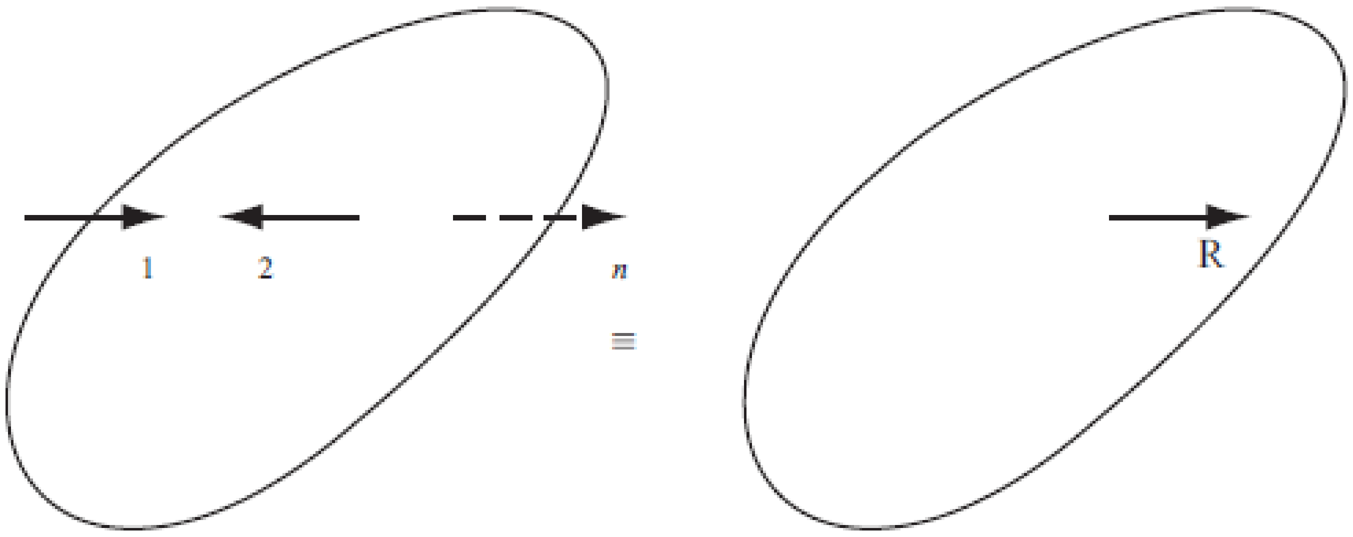
III. Spatial if the lines of action of the forces lie in space

RESULTANT OF A CONCURRENT FORCE SYSTEM

- **Collinear if the forces have the same line of action:** A system of collinear forces acting on a body may be replaced by a single resultant force R acting in the same line of action as the given forces

  where, R = 1 + 2 + 3 + ... N.

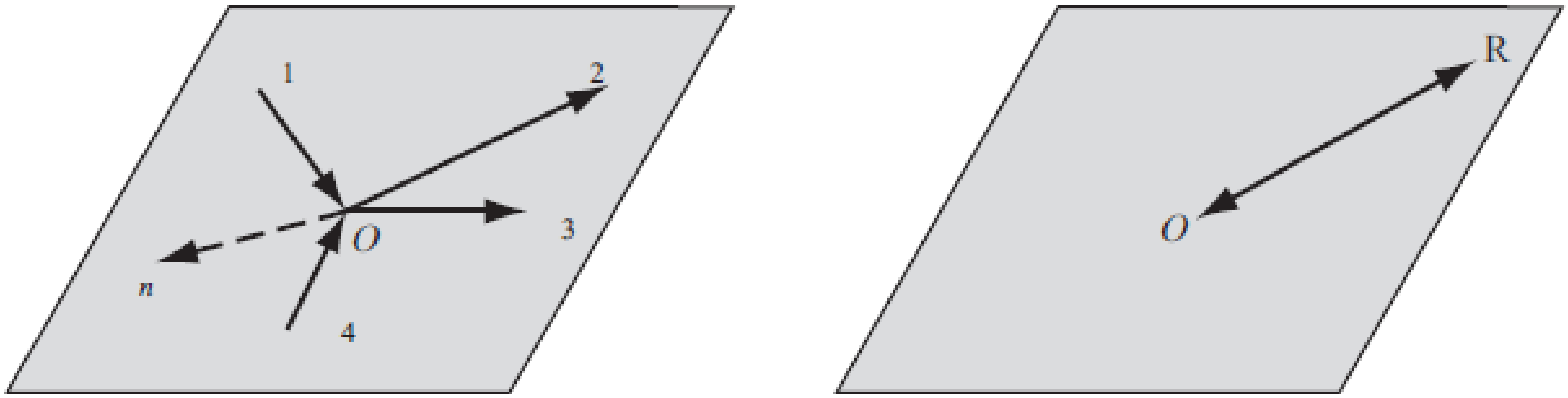
- In other words, the sum of the forces in the collinear force system must provide the resultant.



(a) Collinear forces

RESULTANT OF A CONCURRENT FORCE SYSTEM

- A concurrent force system may be collinear, coplanar or spatial;
  I. Collinear if the forces have the same line of action,
  II. Coplanar if the lines of action of the forces lie in a plane
  III. Spatial if the lines of action of the forces lie in space



(b) Coplanar forces

RESULTANT OF A CONCURRENT FORCE SYSTEM

- **Coplanar if the lines of action of the forces lie in a plane:** A system of coplanar concurrent forces acting on a body may be replaced by a single resultant force R passing through the point of concurrency in that plane
- where, $R = 1 + 2 + 3 + \ldots$
- In terms of rectangular components,

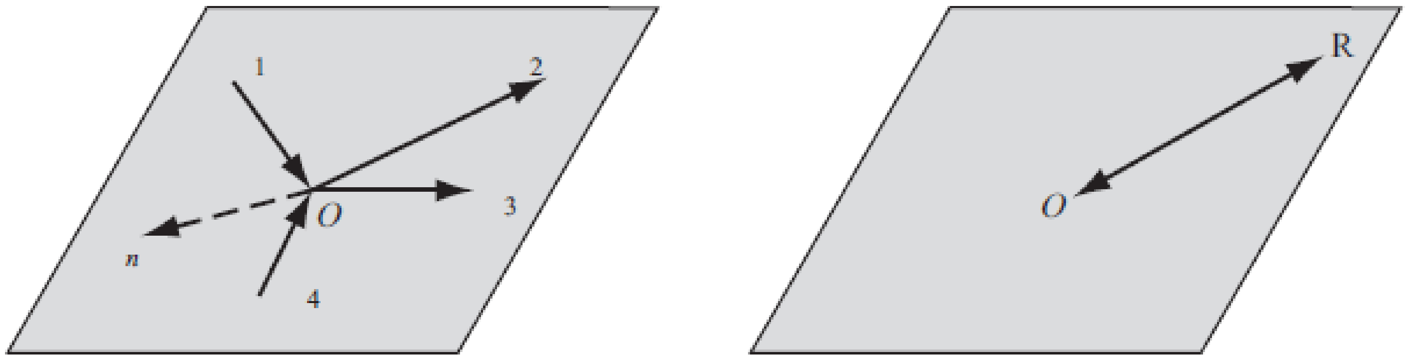
  $1 = F_{1x} + F_{1y}$

  $2 = F_{2x} + F_{2y}$

  $3 = F_{3x} + F_{3y}$ and

  $R = (F_{1x} + F_{2x} + \ldots) + (F_{1y} + F_{2y} + \ldots)$

  $R = R_x + R_y$

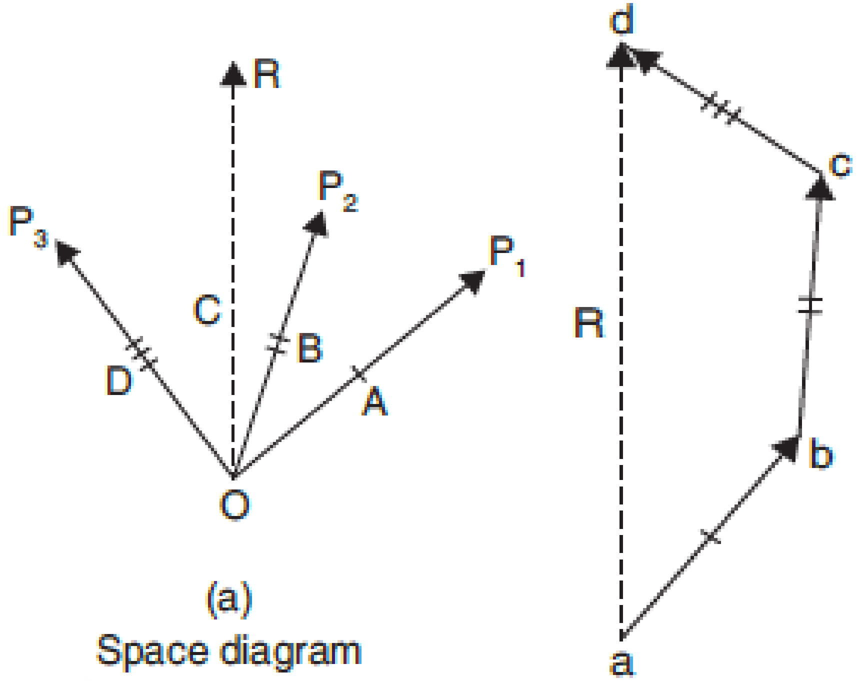


(b) Coplanar forces

- Geometrically, a polygon of forces can be constructed to add the coplanar concurrent forces 1, 2, 3, ... to result in R.

RESULTANT OF SEVERAL COPLANAR CONCURRENT FORCES

- To determine the resultant of a number of coplanar concurrent forces any of the following two methods may be used :
- 1. Graphical method (Polygon law of forces)
- 2. Analytical method (Principle of resolved parts).



(a)
Space diagram

(b)
Vector diagram

Resultant by graphical method. Fig. (*a*) shows the forces *P*1, *P*2 and *P*3 simultaneously acting at a particle *O*.

- Draw a vector *ab* equal to force *P*1 to some suitable scale and parallel to the line of action of *P*1.
- From '*b*' draw vector *bc* to represent force *P*3 in magnitude and direction.
- Now from '*c*' draw vector *cd* equal and parallel to force *P*3.
- Join *ad* which gives the required resultant in magnitude and direction, the direction being *a* to *d* as shown in the vector diagram.

RESULTANT OF SEVERAL COPLANAR CONCURRENT FORCES

- **Resultant by analytical method.** Refer Fig.
- The resolved parts in the direction *OX* and *OY* of

  P_1 are $P_1 \cos \theta_1$ and $P_1 \sin \theta_1$, respectively,

  P_2 are $P_2 \cos \theta_2$ and $P_2 \sin \theta_2$ respectively

  and P_3 and $P_3 \cos \theta_3$ and $P_3 \sin \theta_3$ respectively.

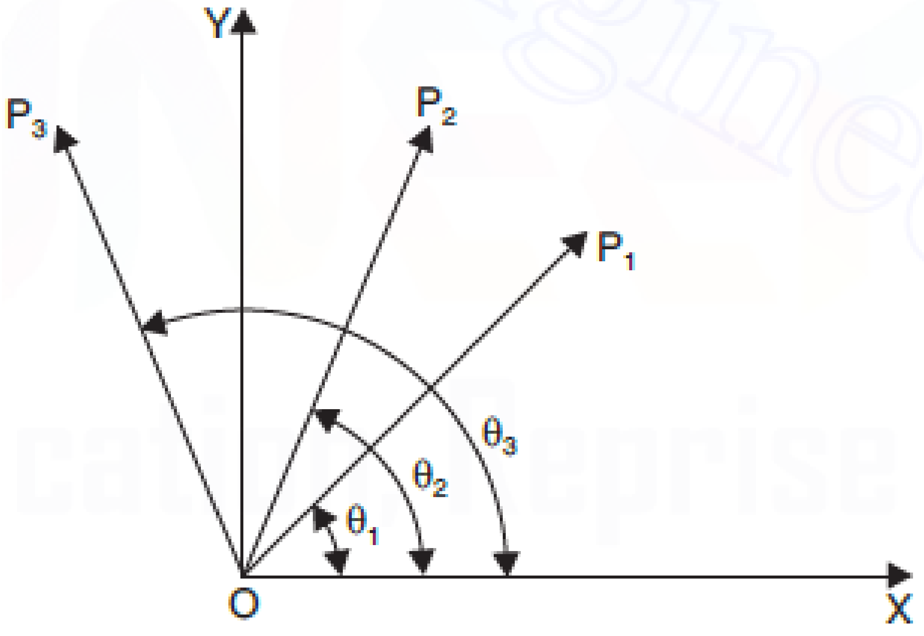
- If the resultant *R* makes an angle θ with *OX* then by the principle of resolved parts :

  $R \cos \theta = P_1 \cos \theta_1 + P_2 \cos \theta_2 + P_3 \cos \theta_3$

  $= \Sigma H$ ... (*i*)

  and $R \sin \theta = P_1 \sin \theta_1 + P2 \sin \theta_2 + P3 \sin \theta_3$

  $= \Sigma V$ ... (*ii*)



RESULTANT OF SEVERAL COPLANAR CONCURRENT FORCES

- Now, by squaring and adding eqns. (*i*) and (*ii*), we get

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

- and by dividing eqn. (*ii*) by eqn. (*i*), we get

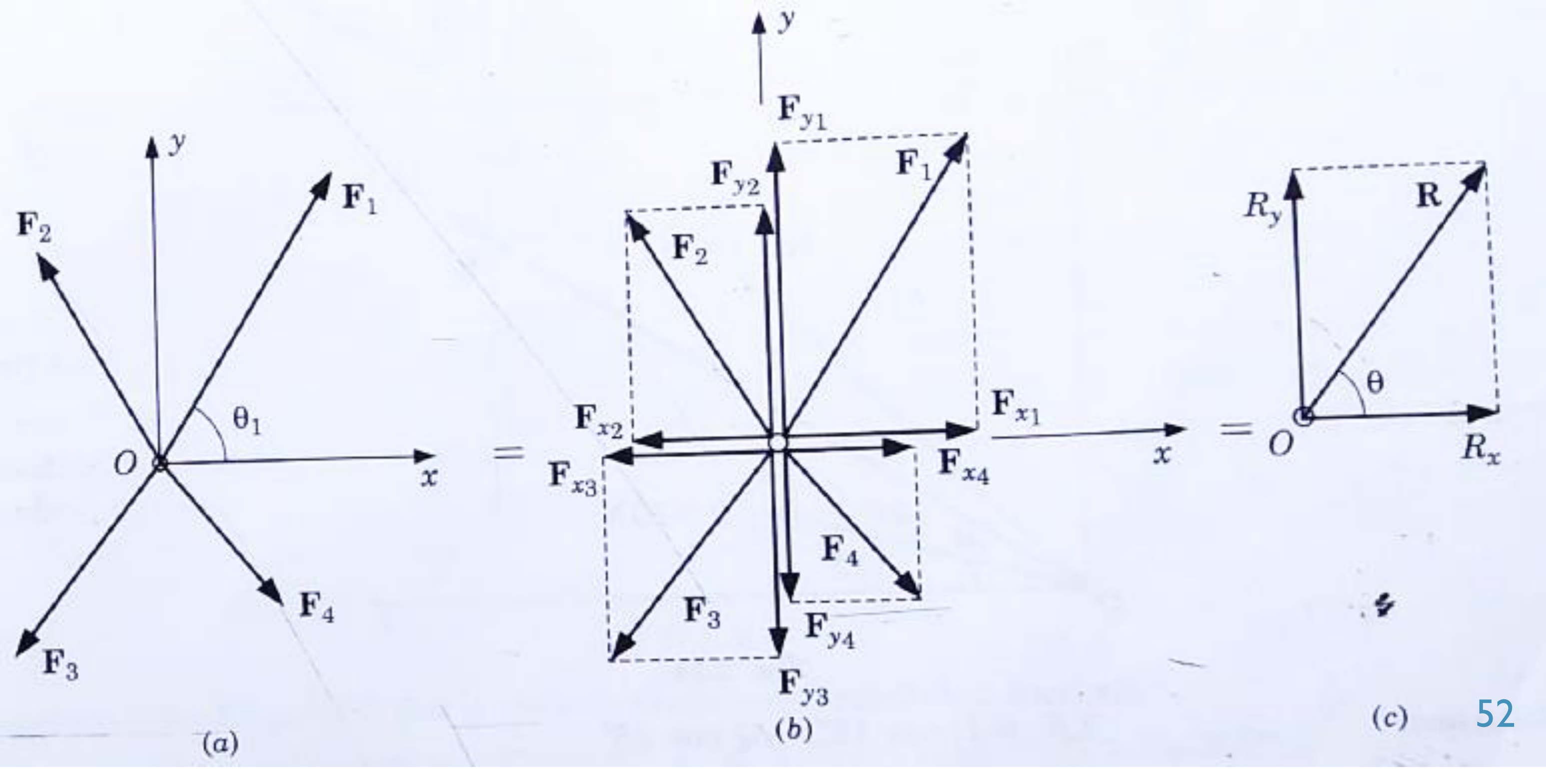
$$\frac{R \sin \theta}{R \cos \theta} = \tan \theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

- It may be noted that while solving problems proper care must be taken about the signs (+ve or –ve) of the resolved parts.
- Following sign conventions may be kept in view :
- **Vertical components** Upward direction ↑ Positive (+) Downward direction ↓ Negative (–)
- **Horizontal components** From left to right → Positive (+) From right to left ← Negative (–)

RESULTANT OF SEVERAL COPLANAR CONCURRENT FORCES

- Resultant of two forces can be found graphically by using parallelogram law or law of triangle of forces. For more than two forces, repeated use of the parallelogram law or the law of polygon of forces can give us the resultant.
- Now let us discuss the analytical method. Consider a number of coplanar forces F_1, F_2, F_3 and F_4 acting on a particle at O.
- For force acting at O can be replaced by its rectangular component Fx_1 Fy_1; Fx_2, Fy_2 etc.
- These components of forces produce the same effect on the particle as the forces themselves.
- Now the horizontal components can be added into a single force Rx by using the law of parallelogram of forces.



RESULTANT OF SEVERAL COPLANAR CONCURRENT FORCES

- This simply reduces to an algebraic sum of horizontal components Fx_1, Fx_2 etc. as they lie along the same line.

  $Rx = Fx_1 + Fx_2 + Fx_3 + Fx_4$

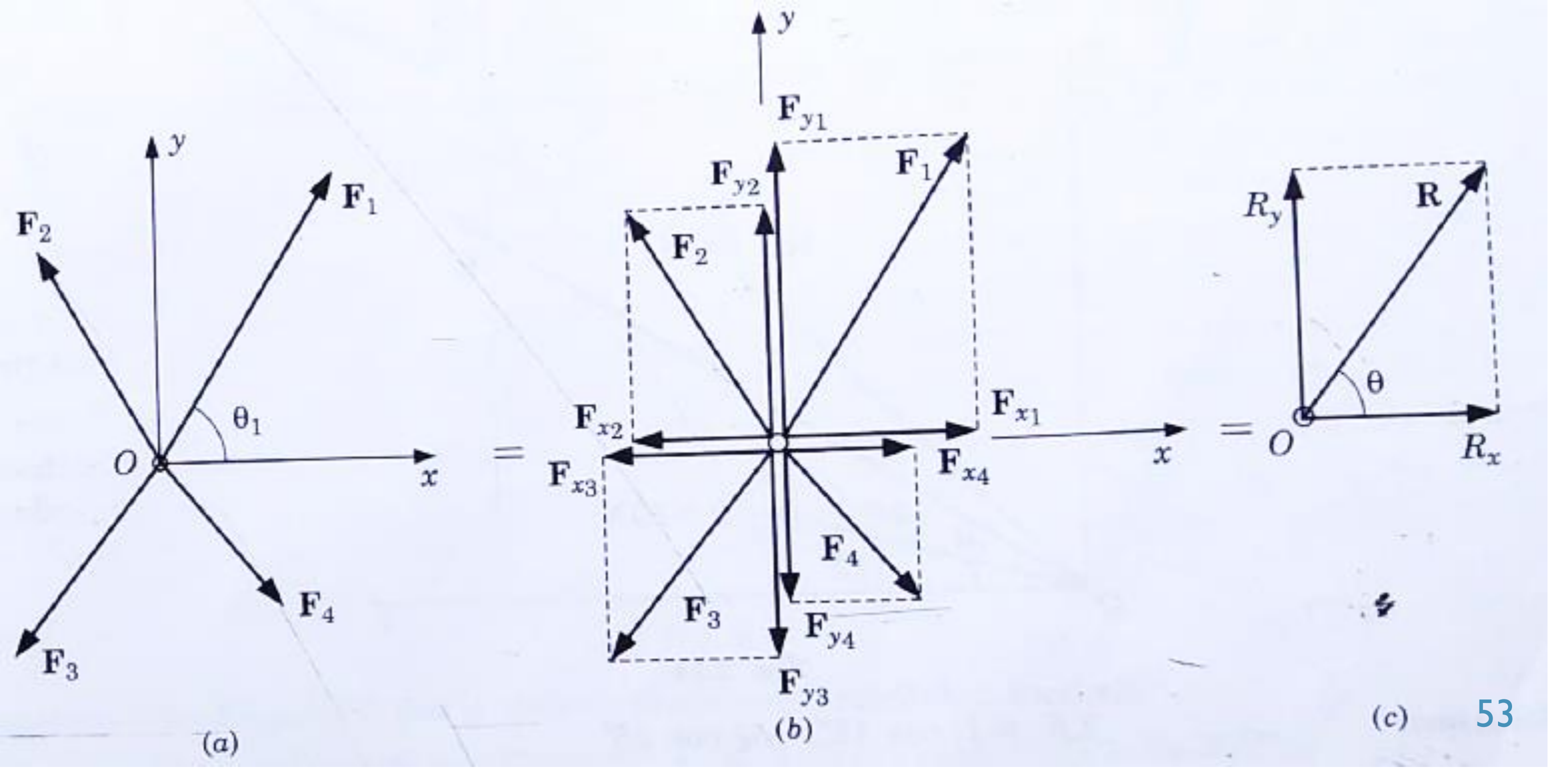
- Similarly, vertical components Fy_1, Fy_2….etc. can be added into a single force Ry such that,

  $Ry = Fy_1 + Fy_2 + Fy_3 + Fy_4$

- Therefore, $Rx = \Sigma Fx$ and $Ry = \Sigma Fy$
- Now, these two perpendicular forces Rx and Ry can be added vectorially to determine resultant R of the forces.

$$R = \sqrt{R_x^2 + R_y^2}$$

$$tan\theta = \frac{R_y}{R_x}$$



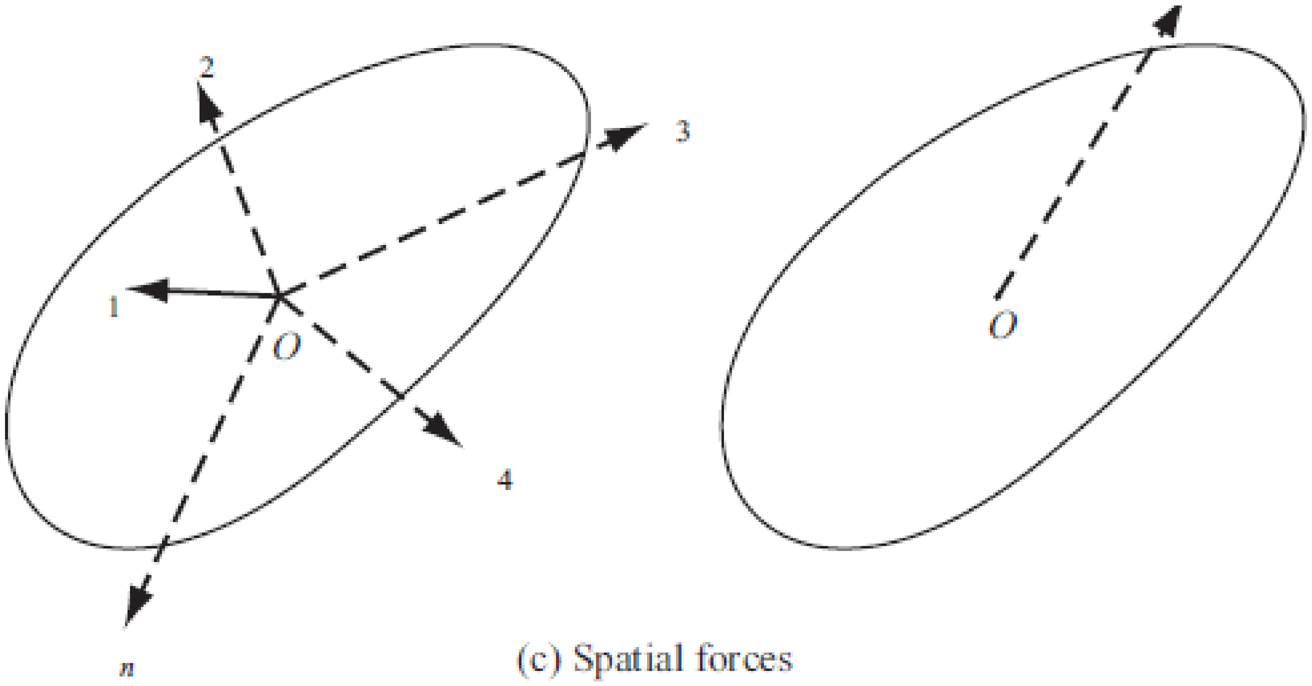
RESULTANT OF A CONCURRENT FORCE SYSTEM

- A concurrent force system may be collinear, coplanar or spatial;

I. Collinear if the forces have the same line of action,

II. Coplanar if the lines of action of the forces lie in a plane

III. Spatial if the lines of action of the forces lie in space



(c) Spatial forces

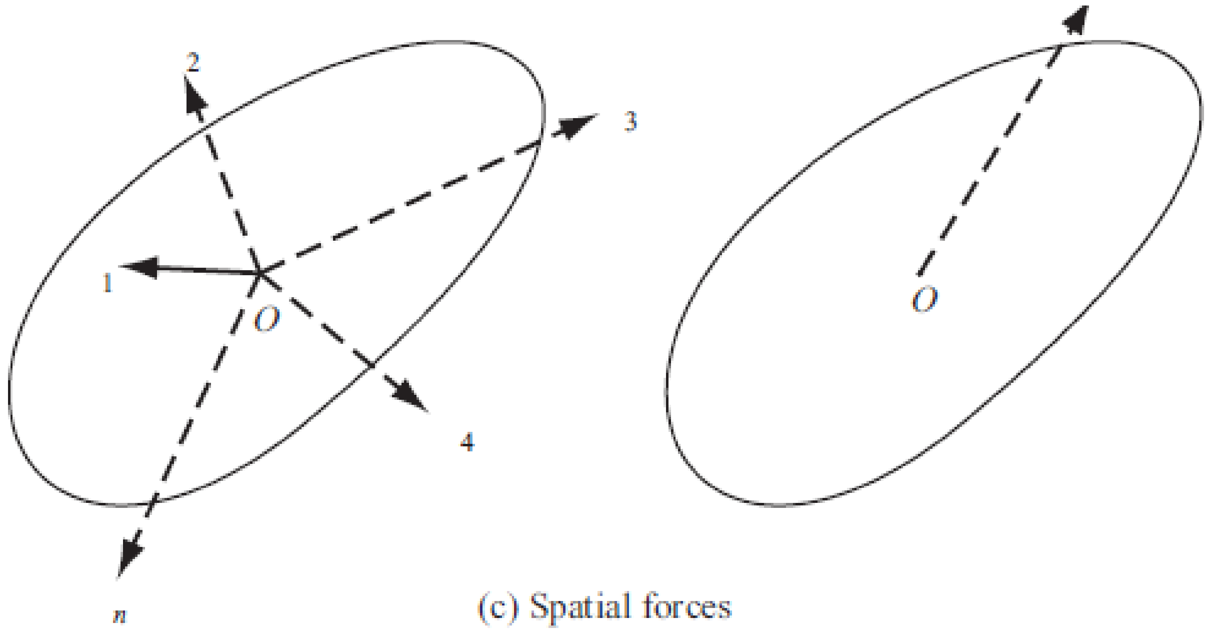
RESULTANT OF A CONCURRENT FORCE SYSTEM

- **Spatial if the lines of action of the forces lie in space:** A system of concurrent forces not confined to a plane, acting on a body may also be replaced by a single resultant force passing through the point of concurrency such that

  $R = 1 + 2 + 3 + \ldots$

  $R = (F_{1x} + F_{2x} + \ldots) + (F_{1y} + F_{2y} + \ldots) + (F_{1z} + F_{2z} + \ldots)$

  $R = R_x + R_y + R_z$



(c) Spatial forces

- In view of the space distribution of the forces, the geometrical construction, though possible is not feasible for adding the spatial forces because of the complexity of drawing space diagrams.

QUESTION 1

Find the magnitude and direction of the resultant of two forces 40 N and 60 N acting at a point with an included angle of 40° between them. The force of 60 N being horizontal.

SOLUTION 1

- **Sol.** Refer to Parallelogram law,
- P = 60 N, Q = 40 N, θ = 40°
- Using the relation,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{(60)^2 + (40)^2 + 2 \times 60 \times 40 \times \cos 40^\circ}$$

$$= \sqrt{3600 + 1600 + 4800 \times 0.766} \quad = \mathbf{94.22\ N.}$$

- Hence **magnitude** of the resultant force **= 94.22 N. (Ans.)**
- $tan\alpha = \frac{Q sin\theta}{P+Q cos\theta} = \frac{40\ sin\ 40^\circ}{60+40\ cos\ 40^\circ} = \frac{25.71}{60+30.64}$ = 0.284

∴ α = 15.85° or 15° 51′

- **Hence the direction of the resultant force = 15° 51′ with the 60 N force. (Ans.)**

QUESTION 2

- *The angle between the two forces of magnitude 20 N and 15 N is 60° ; the 20 N force being horizontal. Determine the resultant in magnitude and direction, if*

(a) the forces are pulls ; and

(b) the 15 N force is a push and 20 N force is a pull.

SOLUTION 2

- **Sol. Case (a)** Refer fig,
- P = 20 N, Q = 15 N, θ = 60°
- Using the relation,

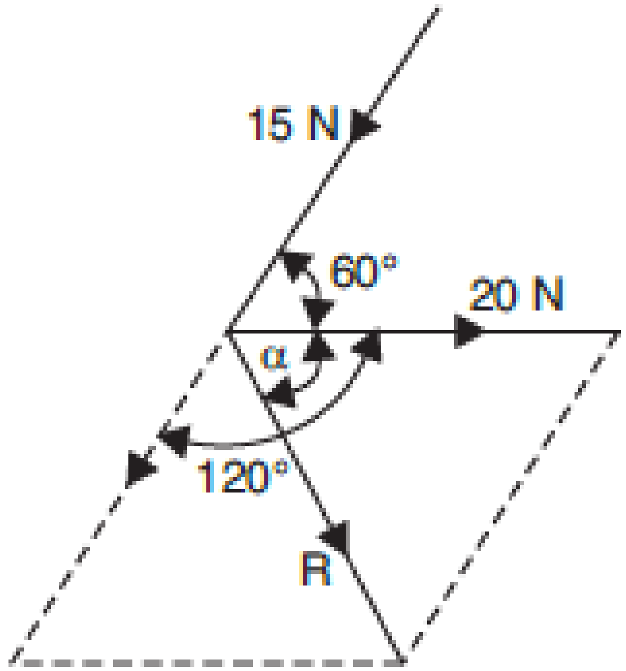
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{(20)^2 + (15)^2 + 2 \times 20 \times 15 \times \cos 60°}$$

$$= \sqrt{400 + 225 + 600 \times 0.5} \quad = \mathbf{30.4\ N.}$$

- Hence **magnitude** of the resultant force **= 30.4 N. (Ans.)**
- $tan\alpha = \frac{Qsin\theta}{P+Qcos\theta} = \frac{15\ sin\ 60°}{20+15\ cos\ 60°} = \frac{12.99}{20+7.5}$

$\therefore \alpha = \tan^{-1} \frac{12.99}{20+7.5}$ = 32.05° or 25° 3′ with 20N force



- **Hence the direction of the resultant force = 25° 3′ with the 20 N force. (Ans.)**

SOLUTION 2

- **Sol. Case (b)** Refer fig,
- P = 20 N, Q = 15 N, θ = 120°
- Using the relation,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{(20)^2 + (15)^2 + 2 \times 20 \times 15 \times \cos 120°}$$

$$= \sqrt{400 + 225 - 300} \qquad = \mathbf{18\ N.}$$

Hence **magnitude** of the resultant force **= 18 N. (Ans.)**

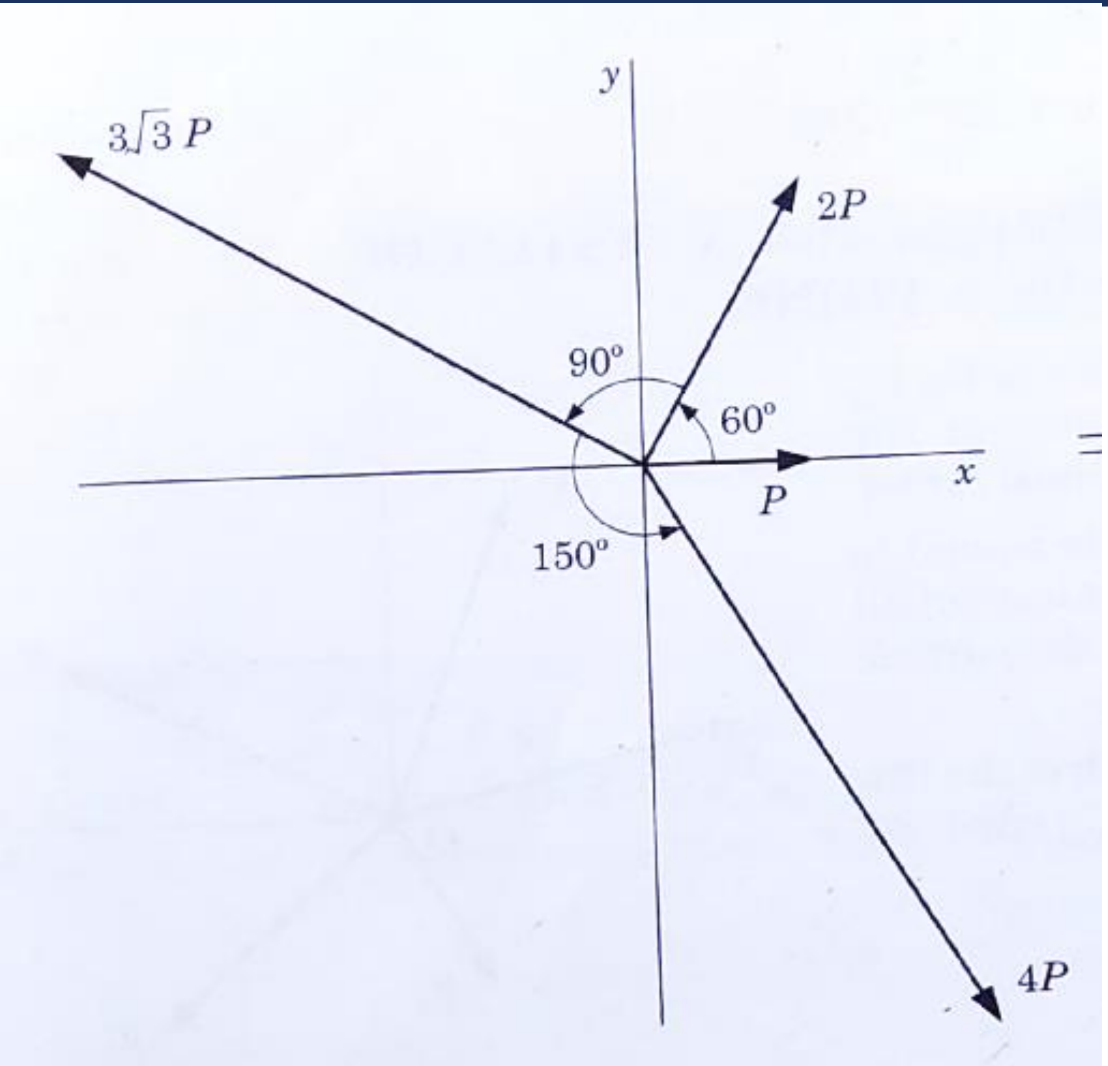
- $\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} = \frac{15\sin 120°}{20+15\cos 120°} = \frac{12.99}{20-15*0.5}$ = 1.039

$\therefore \alpha = \tan^{-1} 1.039 = 46.01° \text{ or } 46°\ 6'$ with 20N force

- **Hence the direction of the resultant force = 46° 6′ with the 20 N force. (Ans.)**

QUESTION 3

- Find the magnitude and direction of the resultant R of four concurrent forces acting as shown in figure.



SOLUTION 3

Solution: Choosing the x and y axes as shown and resolving forces,

$$\Sigma F_x = (P)\cos 0° + (2P)\cos 60° + (3\sqrt{3}\,P)\cos 150° + (4P)\cos 300°$$

$$\Sigma F_x = P + 2P\left(\frac{1}{2}\right) - 3\sqrt{3}P\left(\frac{\sqrt{3}}{2}\right) + 4P\left(\frac{1}{2}\right)$$

$$\Sigma F_x = -\frac{P}{2}$$

$$\Sigma F_y = (P)\sin 0° + (2P)\sin 60° + (3\sqrt{3}\,P)\sin 150° + (4P)\sin 300°$$

$$\Sigma F_y = 0 + 2P\left(\frac{\sqrt{3}}{2}\right) + 3\sqrt{3}P\left(\frac{1}{2}\right) - 4P\left(\frac{\sqrt{3}}{2}\right)$$

$$\Sigma F_y = \frac{\sqrt{3}}{2}P$$

If the resultant of these forces is R having components R_x and R_y,

$$R_x = \Sigma F_x = -\frac{P}{2} \quad \text{and} \quad R_y = \Sigma F_y = \frac{\sqrt{3}}{2}P$$

$$R = \sqrt{R_x^2 + R_y^2} = P\sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

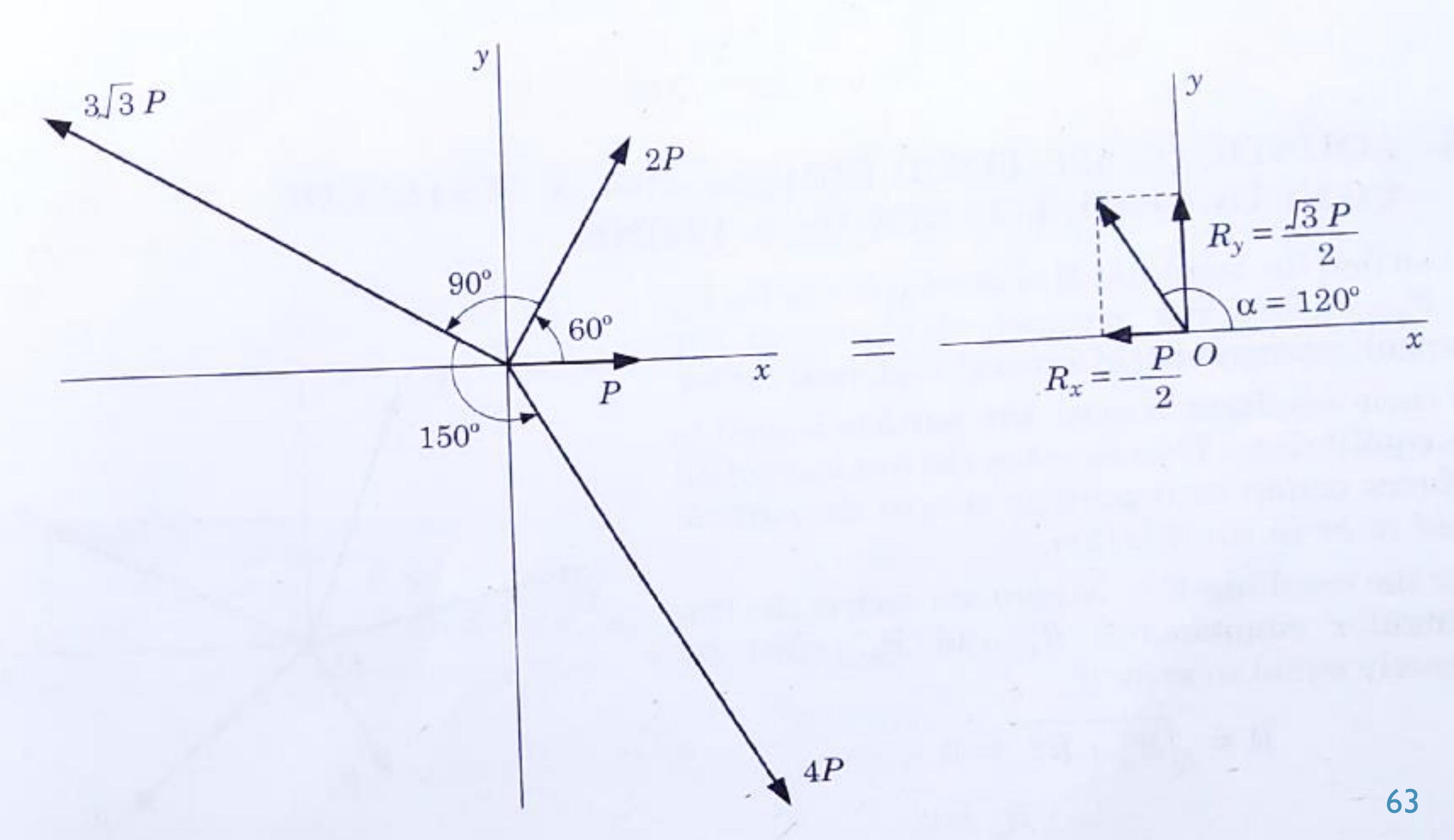
$$R = P \quad Ans.$$

QUESTION 3

- $tan\theta = \frac{R_y}{R_x}$

- $tan\theta = \frac{\frac{\sqrt{3}P}{2}}{\frac{-\sqrt{1}P}{2}}$

- θ = 120 degree



QUESTIONS: MCQS

1. A rigid body is acted upon by a force system. It can in general be brought to equilibrium by the application of a force acting

   (a) on a suitable point on the body

   (b) anywhere along a suitable line

   (c) along a suitable line and a moment along the direction of the force

   (d) along a suitable line and a moment in the direction perpendicular to the direction of force

2. The simplest resultant of a spatial parallel force system is always

   (a) a wrench

   (b) a force

   (c) a moment

   (d) a force and moment

QUESTIONS: MCQS

3. The force of gravitation between two bodies will be inversely proportional to the square of the distance between their centres of masses if the two are

   (a) of constant densities (b) spherical

   (c) of any arbitrary shape (d) of the same shape and size

4. A force acting on a rigid body at a point P can be replaced by a force of equal magnitude and in the same direction at a point Q on the body, together with a moment

   (a) equal in magnitude to PQ times F, acting normal to the plane of and P

   (b) equal in magnitude to F times the distance moved in the lines of actions of the force, acting in the plane of P

   (c) given by × P

SOLUTION: MCQS

1. A rigid body is acted upon by a force system. It can in general be brought to equilibrium by the application of a force acting

   (a) on a suitable point on the body

   (b) anywhere along a suitable line

   (c) **along a suitable line and a moment along direction of force**

   (d) along a suitable line and a moment in the direction perpendicular to the direction of force

2. The simplest resultant of a spatial parallel force system is always

   (a) a wrench

   (b) a force

   (c) **a moment**

   (d) a force and moment

3. The force of gravitation between two bodies will be inversely proportional to the square of the distance between their centres of masses if the two are

   (a) of constant densities

   (b) **spherical**

   (c) of any arbitrary shape

   (d) of the same shape and size

4. A force acting on a rigid body at a point P can be replaced by a force of equal magnitude and in the same direction at a point Q on the body, together with a moment

   (a) equal in magnitude to PQ times F, acting normal to the plane of and P

   (b) equal in magnitude to F times the distance moved in the lines of actions of the force, acting in the plane of P

   (c) **given by × P**

RESULTANT OF TWO LIKE PARALLEL FORCES

- Forces whose lines of action are parallel are called parallel forces.
- They are said to be like when they act in the same sense, they are said to be unlike when they act in opposite sense.
- Let the like parallel forces P and Q act at the points A and B and let their resultant R cut AB at C.
- By resolving parallel to P or Q we find that $R = P + Q$ and that R is parallel to P and Q.
- The moment of R about C is zero, so that the algebraic sum of moments of P and Q about C must also be zero.
- Through C draw MN perpendicular to P and Q then

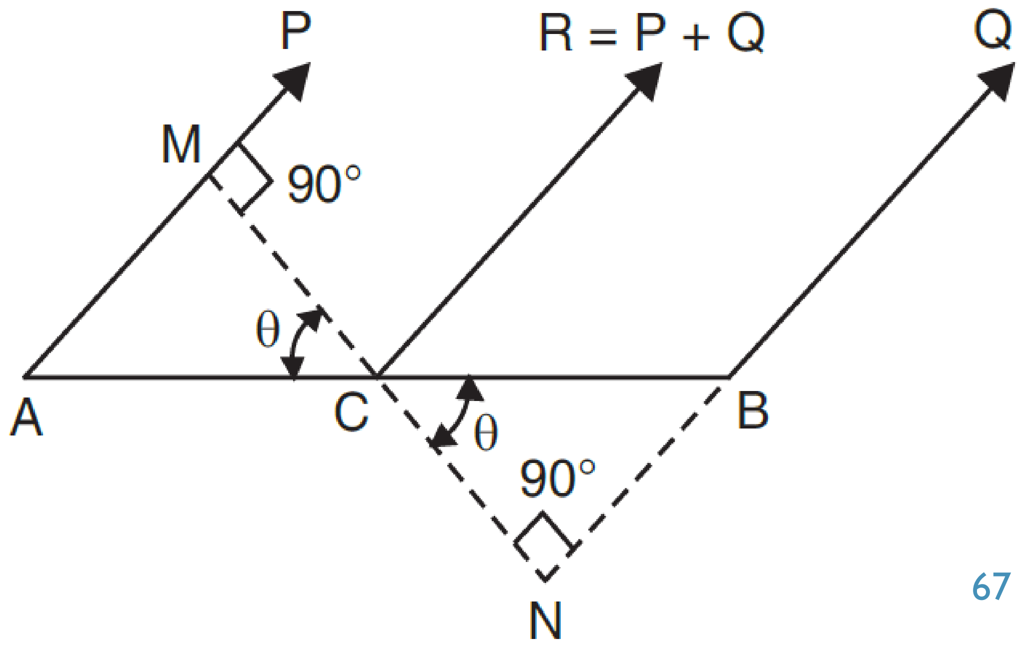
  $CM = AC \cos\theta$ and $CN = BC \cos\theta$
- Taking moments about C,

$P \times CM = Q \times CN$

$P \times AC \cos\theta = Q \times BC \cos\theta$ ($CM = AC \cos\theta$ and $CN = BC \cos\theta$)

$P \times AC = Q \times BC$ *or* $P/BC = Q/AC$

$\therefore$ C divides AB internally in the inverse ratio of the forces.



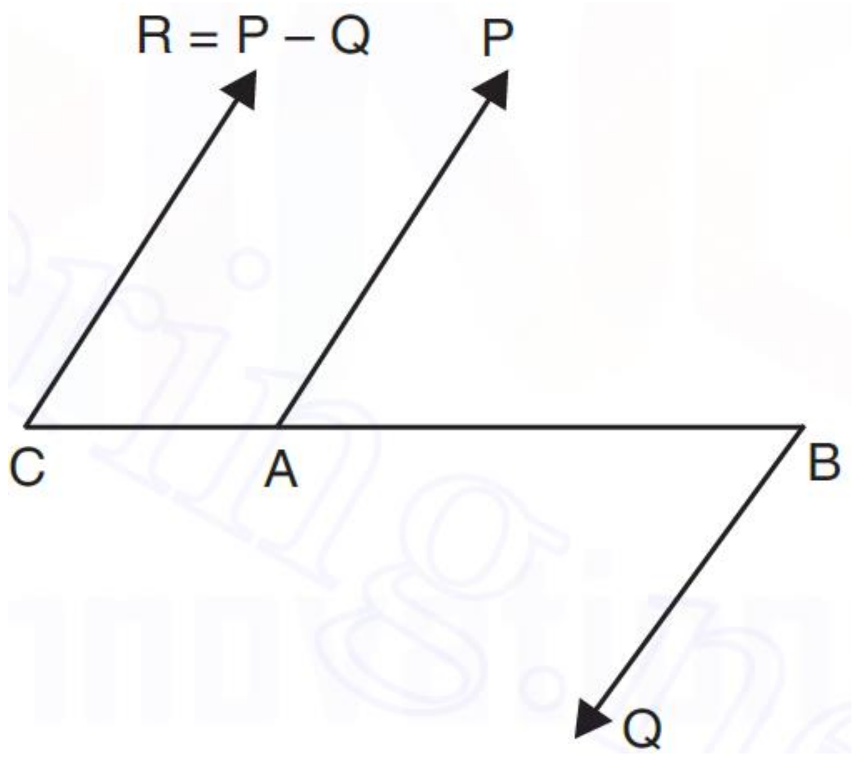
RESULTANT OF TWO UNLIKE PARALLEL FORCES

- Let two unlike parallel forces *P* and *Q* act at *A* and *B*, and let their resultant *R* meet *AB* at *C*. Let *P* be greater than *Q*.
- By resolving parallel to *P* or *Q*, we get $R = P - Q$ acting in the same sense as *P*.
- The algebraic sum of the moments of *P* and *Q* about *C* must be zero so that these moments must be equal and opposite.
- Hence *C* must lie outside *AB*, and it must be nearer to *A* than to *B*.
- Taking moments about *C*, we get

  $P \times AC = Q \times BC$

  or $P/BC = Q/AC$

  ∴ *C* divides *AB* externally in the inverse ratio of the forces.
- The point *C* is called the *centre of parallel forces*.
- It is clear that the *position of C is independent of the inclination of the forces to AB*.



CONCLUSION: RESULTANT OF PARALLEL FORCES

- The resultant of two like parallel forces is equal to the sum of the two parallel forces and it acts in a direction parallel to them.
- Analytically, the resultant of two or more parallel forces can be determined by the method of summing up their x and y components.
- The position of the line of action of the resultant then can be determined by using theorem of Varignon.
- In case of two unlike parallel forces, the resultant lies outside the line joining the points of action of the two forces and on the same side as the larger force.

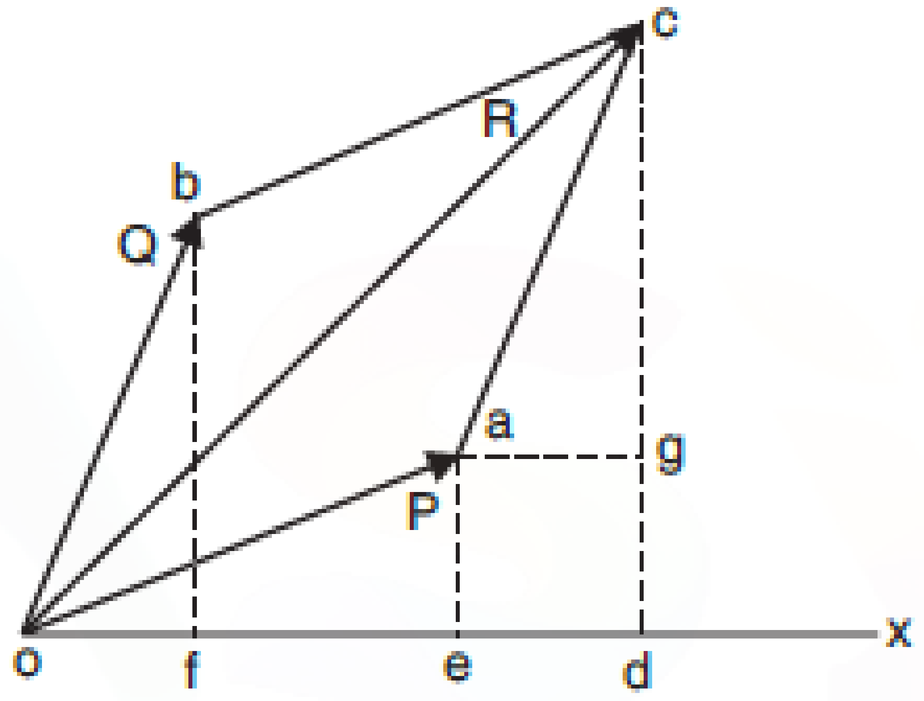
PRINCIPLE OF RESOLVED PARTS

- It states :"The sum of the resolved parts of two forces acting at a point in any given direction is equal to the resolved parts of their resultant in that direction.
- Refer Fig. Let the two forces P and Q be represented by the sides oa and ob of the parallelogram oacb and the resultant R of these two forces is given by the diagonal oc in magnitude and direction.
- Let ox is the given direction.
- Draw bf, ae, cd and ag perpendicular to cd.
- Now from the two triangles obf and acg which are same in all respects, we get

$$of = ag = ed$$

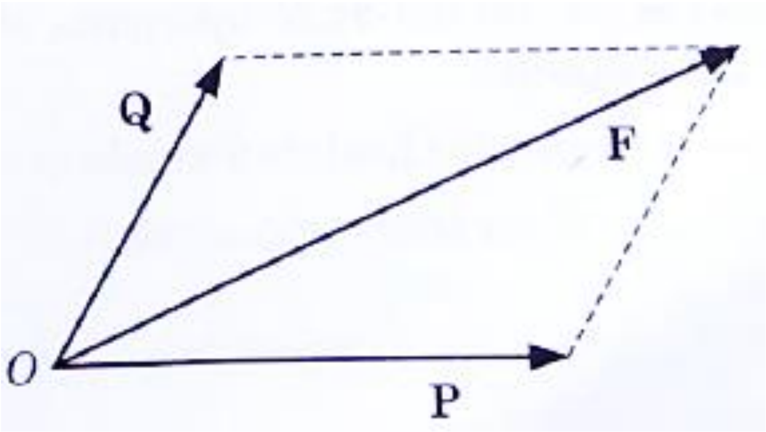
$$\therefore od = oe + ed = oe + of$$

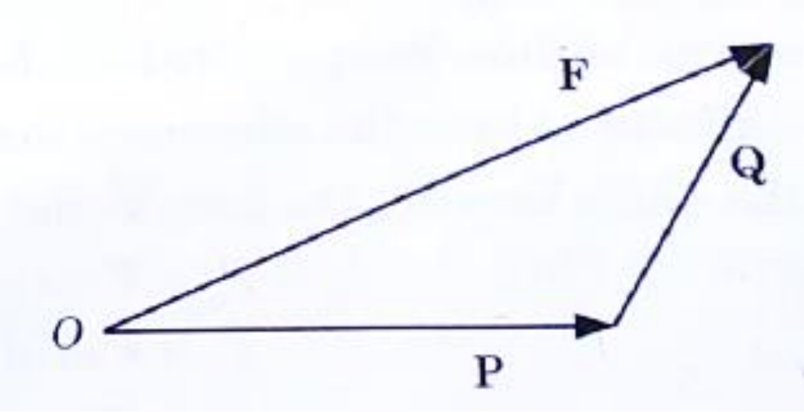
- But oe, of and od represent the resolved components or parts of the forces P, Q and R respectively in the direction of ox.
- It may be noted that this principle holds good for any number of forces



RESOLUTION OF FORCE INTO COMPONENTS

- We have learnt to determine the resultant of two or more forces acting on a particle. Let us now consider the reverse problem.
- That is, how to replace a single force F acting on a particle by two or more forces which (together) have the same effect on the particle as the force F?
- These forces are called the components of the original force F and the process is called resolving the force **F** into components.
- Theoretically, a force can be resolved into an infinite number of sets of components of forces. In practice, a situation requiring the resolution of a force F into two components often arises.





RESOLUTION OF FORCE INTO COMPONENTS

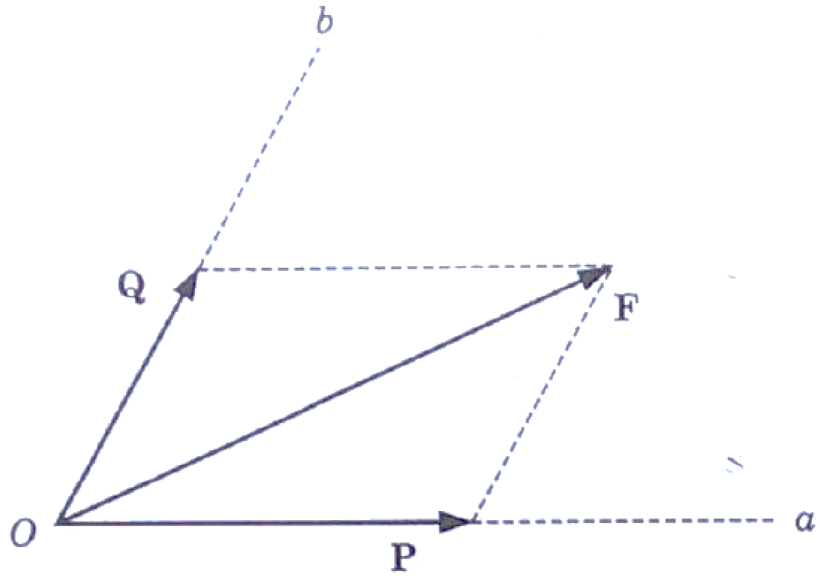
- Two such cases are discussed below:

a. One component P known is magnitude and direction and the other component Q is to be determined.

   - The second component Q of force F can be obtained graphically either by completing the parallelogram or by completing the triangle as shown in Figure.

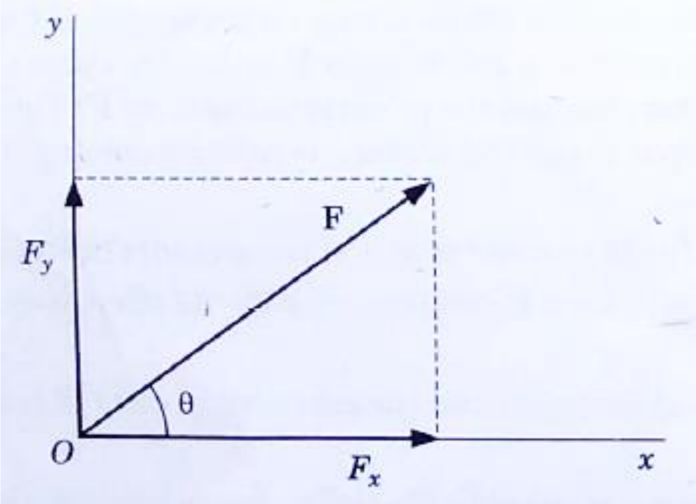
b. When the lines of action of both the components are known but their magnitudes P and Q are to be determined.

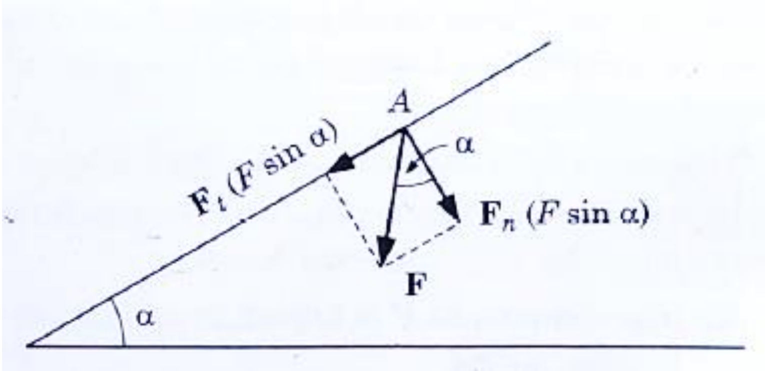
   - The magnitudes P and Q of the two components can be determined by applying the parallelogram law. From the head of the vector F, draw two lines parallel to the given lines of action (Oa and Ob) of the two forces as shown. This shall define the components P and Q of the force F graphically as shown.



RESOLVING A FORCE INTO RECTANGULAR COMPONENTS

- Often it is required to resolve a given force into components which are perpendicular to each other. Such components are called rectangular components.
- Consider a force F which is to be resolved into rectangular components along the x and y axis.
- The component, Fx along the x-axis and the component Fy along the y-axis are obtained using the parallelogram law and completing the rectangle as shown in figure 1.
- The x and y axes are generally horizontal and vertical. But any other directions can also be chosen.
- For example, a vertical force F can be resolved into components tangential and perpendicular to an inclined plane as shown in figure 2.



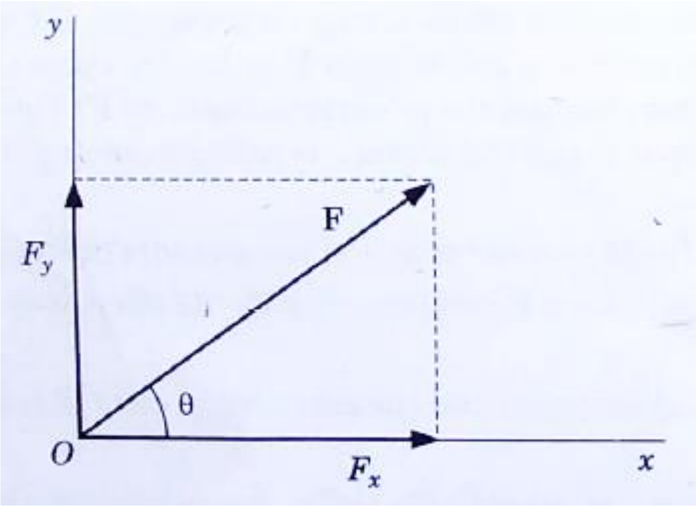


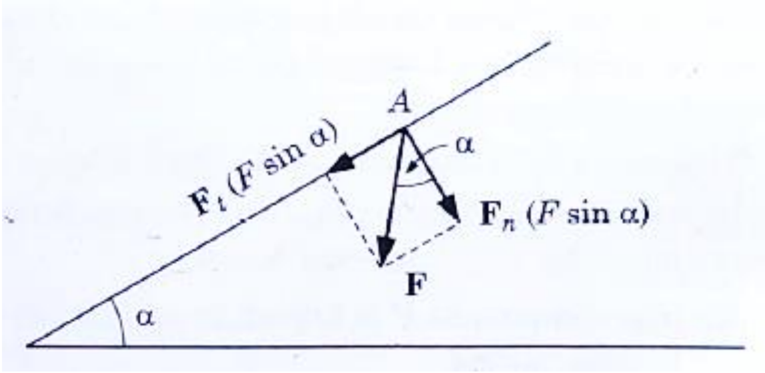
RESOLVING A FORCE INTO RECTANGULAR COMPONENTS

- The rectangular components Fx and Fy are also called the scalar components of the force F as the directions of these components have been predefined.
- These components have the advantage that they can be manipulated algebraically. If θ be the angle between the force F and the x-axis then from trigonometry,

Fx = F cos θ and Fy = F sin θ

- $tan\theta = \frac{Fx}{Fy}$
- $F = \sqrt{F_x^2 + F_y^2}$





FILL IN THE BLANKS :

(*i*) ...... is something which changes or tends to change the state of rest or of uniform motion of a body in a straight line.

(*ii*) Magnitude, direction, sense and ...... are the characteristics of a force.

(*iii*) The resistance to deformation, or change of shape, exerted by the material of body is called an ...... force.

(*iv*) A force which prevents the motion, deformation of body is called a ...... force.

(*v*) An ...... force is one which causes a body to move or change its shape.

(*ii*) Forces whose lines of action pass through a common point are called ...... forces.

(*vii*) A ...... is a single force which can replace two or more forces and produce the same effect on the body as the forces.

(*viii*) A body whose dimensions are practically negligible is called a ......

(ix) ...... units are used by engineers for all practical purposes.

(*x*) …….. states that when a force acts upon a body its effect is the same whatever point in its line of action is taken as the point of application provided that the point is connected with the rest of the body in the same invariable manner.

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Answers 1. (*i*) Force (*ii*) point of application (*iii*) internal (*iv*) passive (*v*) active (*vi*) concurrent (*vii*) resultant force (*viii*) particle (ix) SI (*x*) transmissibility principle

SAY 'YES' OR 'NO' :

(*i*) A force system is a collection of forces acting on a body in one or more planes.

(*ii*) Coplanar concurrent collinear force system includes those forces whose vectors do not lie along the same straight line.

(*iii*) The force whose point of application is so small that it may be considered as a point is called a concentrated force.

(*iv*) A distributed force is one whose place of application is a point.

(*v*) The method of determination of the resultant of some forces acting simultaneously on a particle is called composition of forces.

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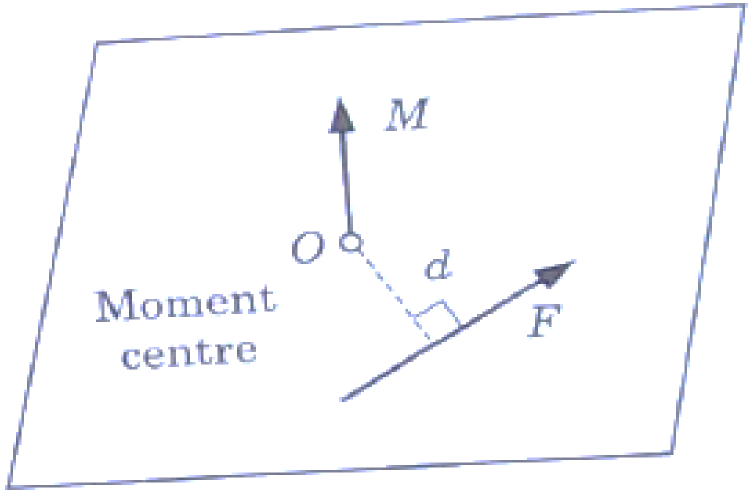
Answers (*i*) Yes (*ii*) No (*iii*) Yes (*iv*) No (*v*) Yes.

MOMENT OF A FORCE

- A force can rotate a nut when applied by a wrench or can open a door while the door rotates on its hinges. A force thus, can produce a rotary motion besides producing a translatory motion.
- The measure of this turning effect produced by a force on a body can be called as the **moment of force. Or** This rotational tendency of a force is called **moment.**
- **Moment of a force about an axis:** *The moment of a force about an axis through a point, or for short the moment of a force about a point is equal to the product of the force and perpendicular distance of the point from the line of action of the force.*

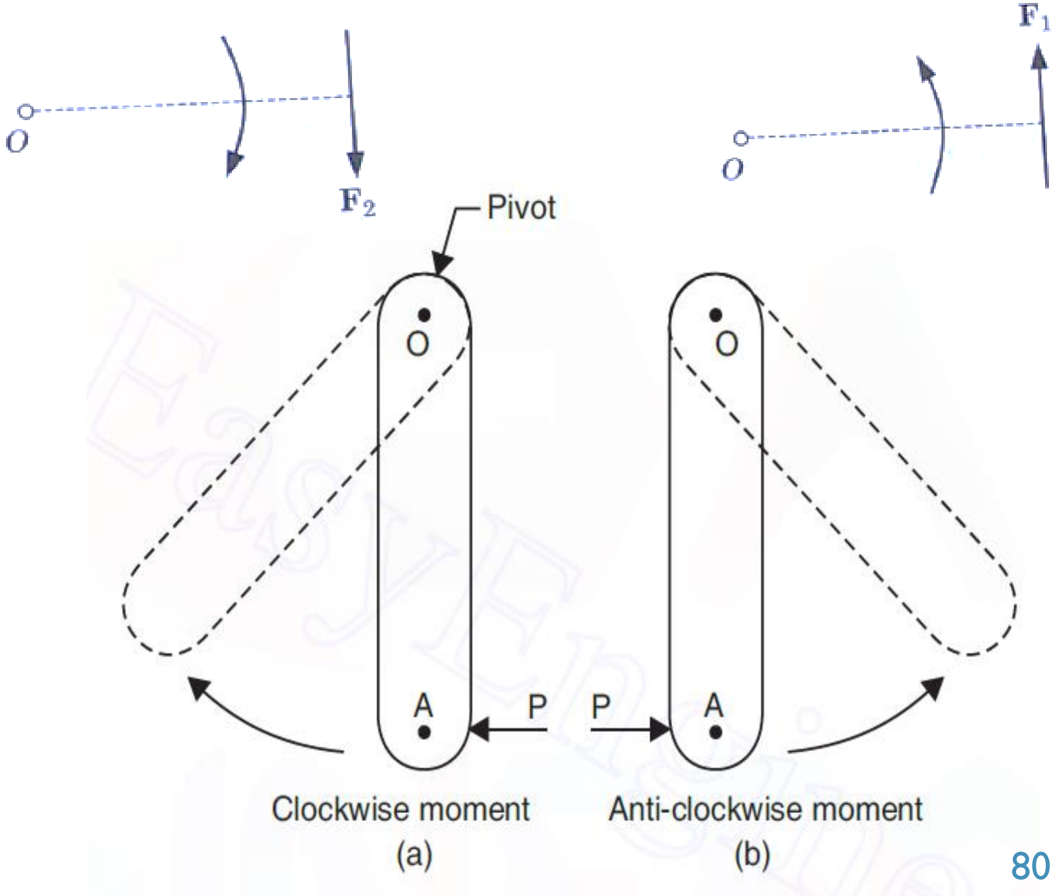


$$M_o = F * d$$

- The point o is called the moment centre and the distance d is called the arm of the force.
- The unit of the moment = Unit of the force * unit of the distance. [Newton-metre (N-m)]

MOMENT OF A FORCE

- The moment of a force about a point is a vector which is directed perpendicular to the plane containing the moment centre and the force.
- But usually we are concerned with the direction of its rotational tendency that is whether an applied force tends to rotate a body clockwise or anticlockwise.
- In the figure (a), the force F_1 has a tendency to produce a clockwise rotation about the moment centre.
- In the figure (b) the force F_2 has a tendency to produce an anticlockwise rotation about the moment centre.
- As a matter of convention, an anticlockwise moment is taken to be positive and clockwise moment as negative.
- While adding moments, the sense of each moment should be taken into account.

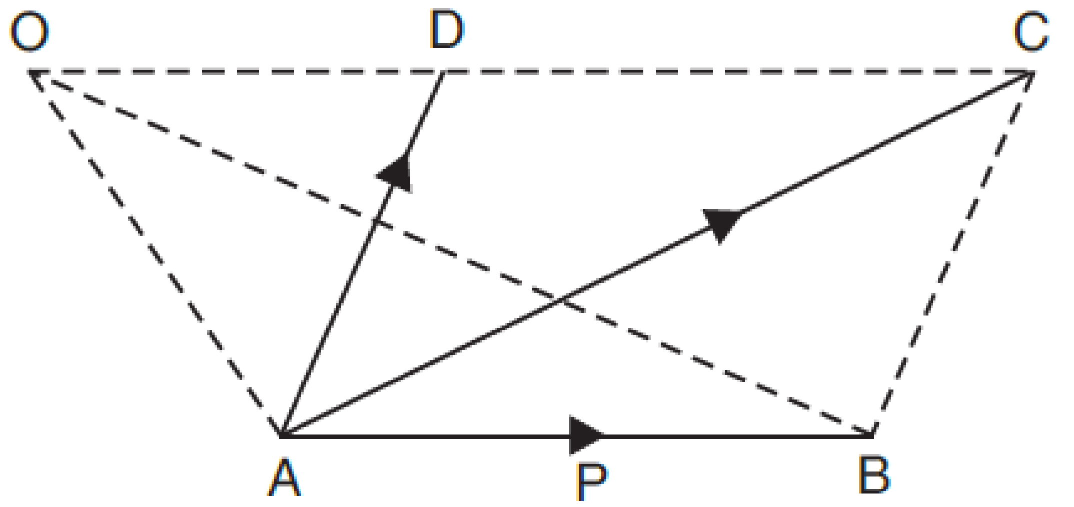


VARIGNON'S THEOREM:

- **Varignon's theorem** states that the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.
- **The principal of moments** states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.
- Varignon's theorem in mechanics establishes the dependence between moments of forces of a given system and moment of their resultant forces.
- This theorem was first formulated and proved by the French scientist P. Varignon.
- *Varignon's theorem is applicable to cases, where the two forces give out a single resultant and it does not apply when the forces form a couple since the resultant force of a couple is zero.*
- The principle of moments is the extension of Varignon's theorem.

VARIGNON'S THEOREM: CASE 1. *WHEN THE TWO FORCES MEET AT A POINT*

- Figure shows two forces *P* and *Q* acting at *A*.
- The magnitude of *P* is represented by *AB* and that of *Q* is represented by *AD*. Complete the parallelogram *ABCD*.
- *AC* represents the resultant *R* of *P* and *Q*.
- Take any other point *O* in the plane of the forces *P* and *Q* and in the line *CD* produced as shown.
- Join *OB* and *OA*.



Moment of *P* about *O* = 2 ΔOAB

Moment of *Q* about *O* = 2 ΔOAD

Moment of *R* about *O* = 2 ΔOAC

But area of ΔOAB = area of ΔABC = area of ΔACD

- Adding algebraically the moments of *P* and *Q*

VARIGNON'S THEOREM: CASE 1. *WHEN THE TWO FORCES MEET AT A POINT*

- Adding algebraically the moments of P and Q

$= 2\,\Delta OAB + 2\,\Delta OAD$

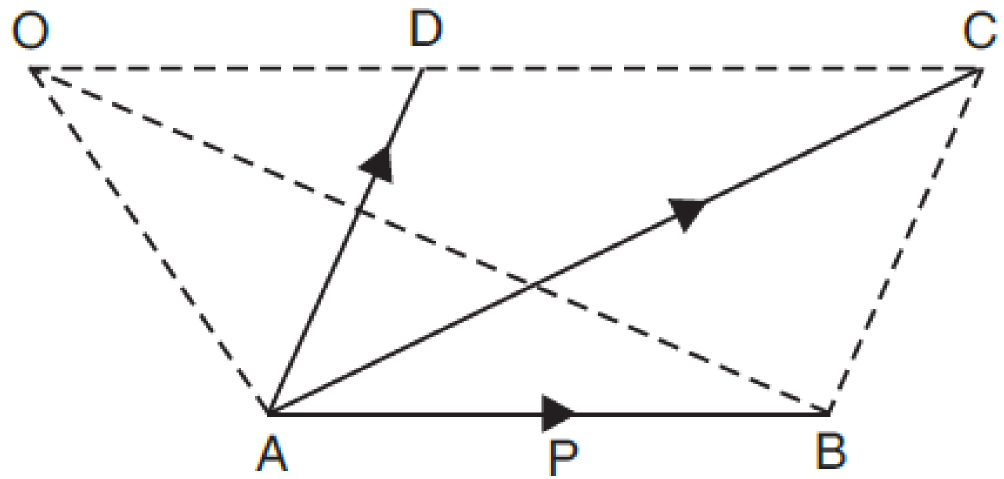
$= 2\,\Delta ACD + 2\,\Delta OAD$

(Substituting ΔACD for ΔOAB)

$= 2\,(\Delta ACD + \Delta OAD)$

$= 2\,\Delta OAC$

= Moment of R about O.

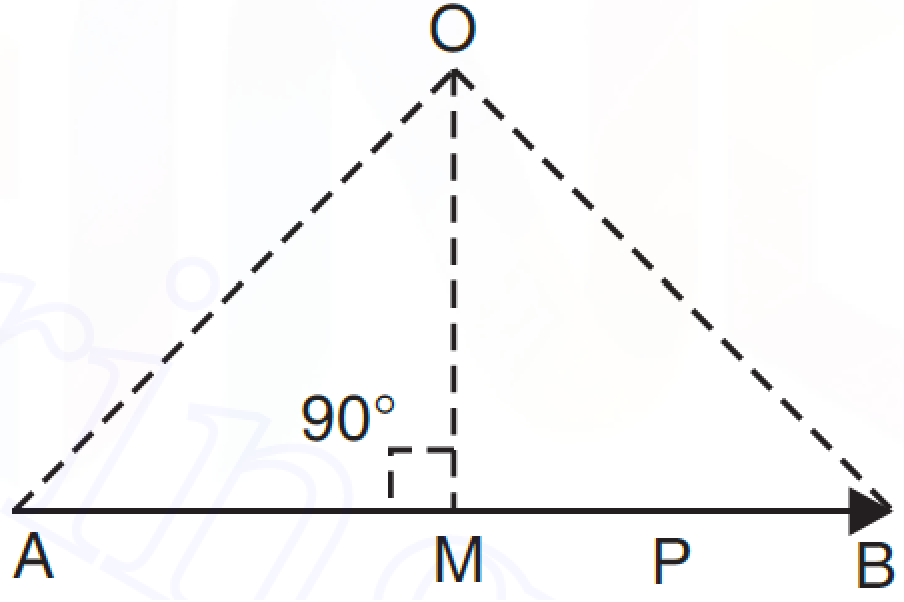


VARIGNON'S THEOREM: CASE 1. *WHEN THE TWO FORCES MEET AT A POINT*

- Consider a force *P* which can be represented in magnitude and direction by the line *AB*.
- Let *O* be the point, about which the moment of this force is required.
- Draw *OM* perpendicular to *AB* and join *OA* and *OB*.
- Now moment of force *P* about *O* = $P \times OM$

  $= AB \times OM$
- But $AB \times OM$ is equal to twice the area of triangle *OAB* because geometrically the area of this triangle is equal to $(AB \times OM)/2$.
- ∴ Moment of force *P* about *O* *i.e.*,

  $AB \times OM = 2\,\Delta AOB$.



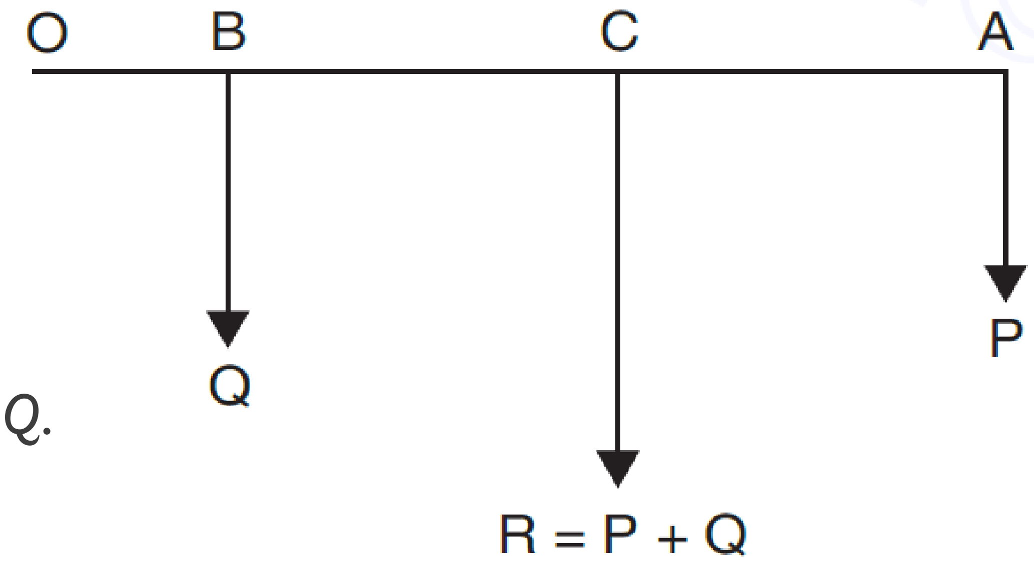
VARIGNON'S THEOREM: CASE 2. *WHEN THE TWO FORCES ARE PARALLEL TO EACH OTHER*

- Let P and Q be the two parallel forces as shown in Figure.
- Draw a line AB perpendicular to the forces to meet their lines of action in A and B.
- Locate any point O in the plane of the two forces on AB produced.
- Resultant of P & Q is R which is equal to sum of forces P & Q.
- Let it act through a point C in AB so that $Q \times CB = P \times CA$
- The sum of moments of P and Q about O

$= P \times OA + Q \times OB = P(OC + CA) + Q(OC - CB)$

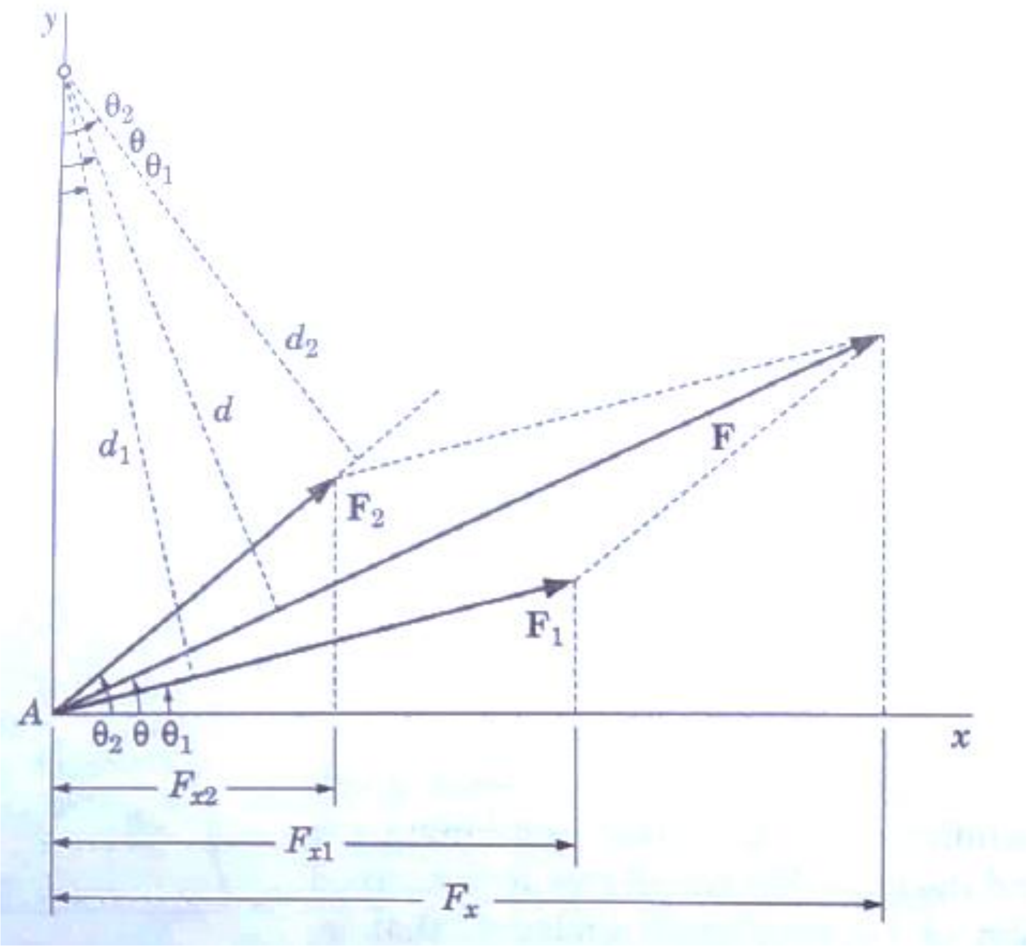
$= (P + Q)\, OC + P \times CA - Q \times CB$

$= (P + Q)\, OC \; (\because Q \times CB = P \times CA)$ = Moment of R about O.



VARIGNON'S THEOREM:

- Consider a force F acting at a point A and having components F_1 and F_2 in any two directions.
- Let us choose any point O, lying in the plane of the forces, as a moment centre.
- Attach at 'A' two rectangular axes such that the y-axis is along the line AO and the x-axis is perpendicular to it as shown in figure.



VARIGNON'S THEOREM:

- Moment of the force F about O,

  $Fd = F(OA\cos\theta) = OA\,(F\cos\theta)$

  $Fd = OAF_x$………………… …………………………..1

- Moment of the force F_1 about O,

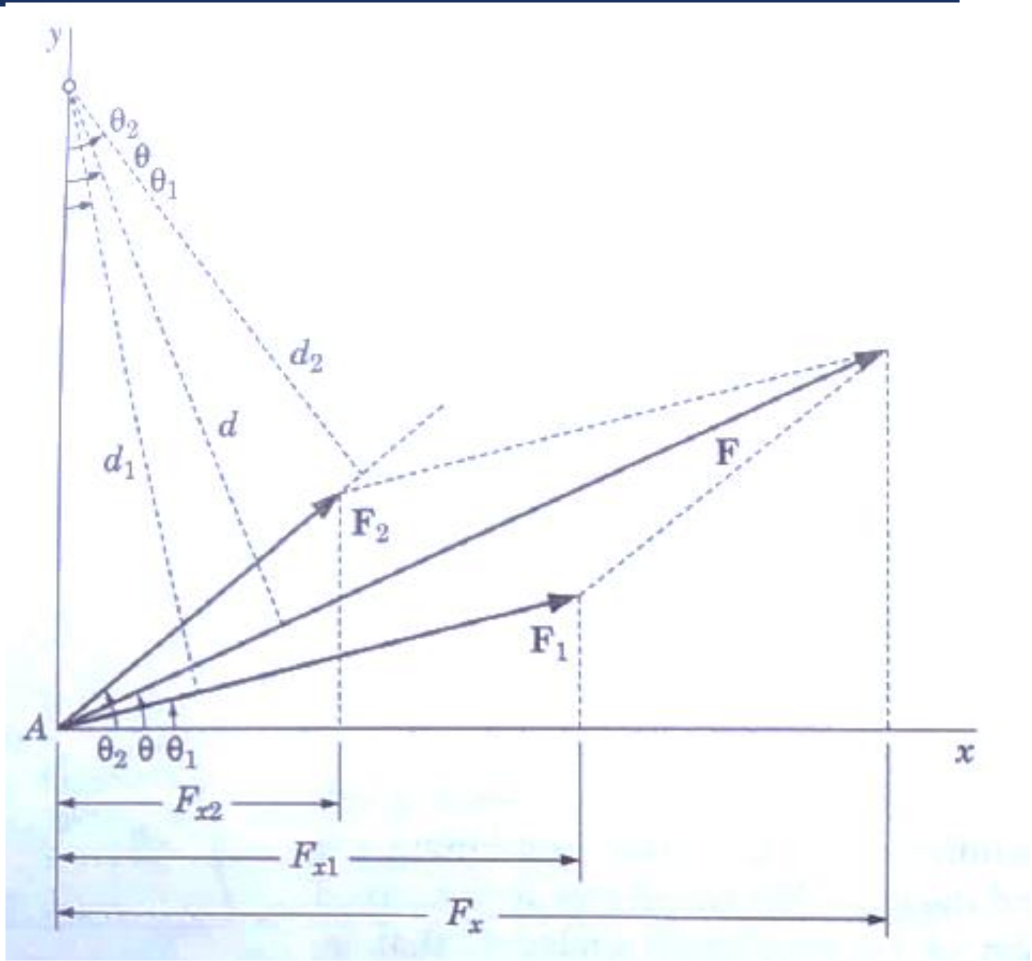
  $F_1 d_1 = F_1(OA\cos\theta_1) = OA\,(F_1\cos\theta_1)$

  $F_1 d_1 = OAF_{x1}$…………………..........…………………….2

- Moment of the force F_2 about O,

  $F_2 d_2 = F_2(OA\cos\theta_2) = OA\,(F_2\cos\theta_2)$

  $F_2 d_2 = OAF_{x2}$…………...3



VARIGNON'S THEOREM:

- Adding equations 2 and 3,

$F_1d_1 + F_2d_2 = OAF_{x1} + OAF_{x2}$

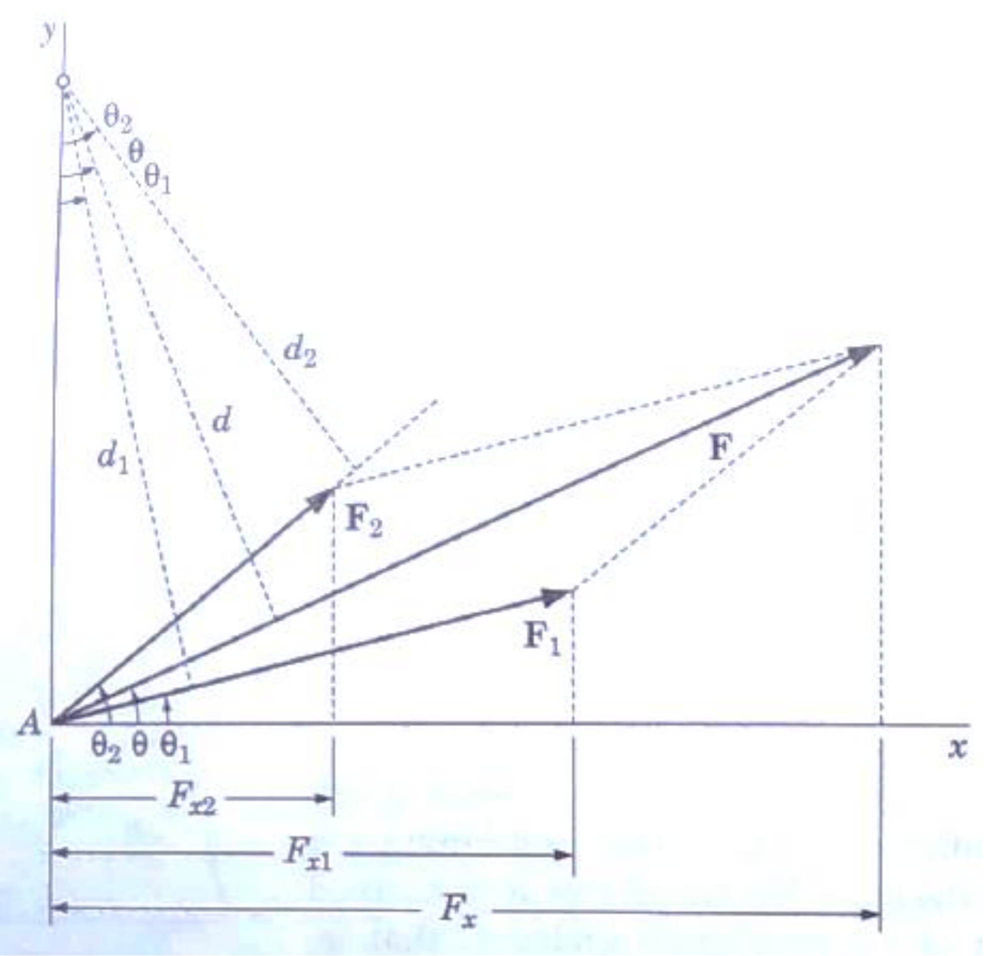
$F_1d_1 + F_2d_2 = OA\,(F_{x1} + F_{x2})$

- But, $F_x = F_{x1} + F_{x2}$ as, sum of the x-components of forces F_1 and F_2= x-components of the resultant Force F

Therefore, $OA\,(F_x) = OA\,(F_{x1} + F_{x2})$

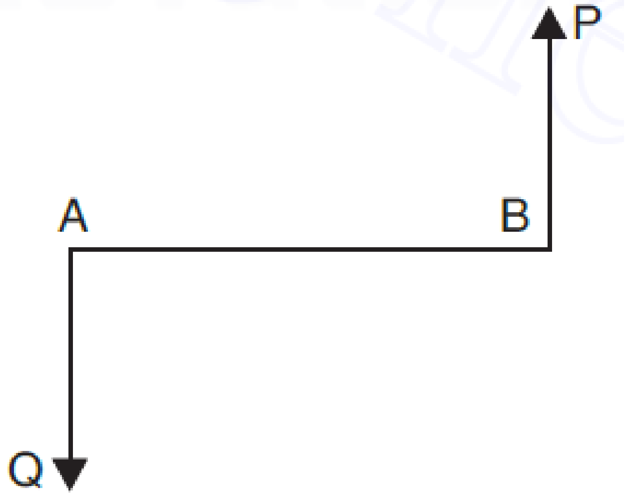
So, $Fd= F_1d_1 + F_2d_2$

- By successive application of the above method, the theorem of Varignon can be extended to a system of several forces and their resultant



COUPLE

- *A couple is pair of two equal and opposite forces acting on a body in a such a way that the lines of action of the two forces are not in the same straight line.*
- The effect of a couple acting on a rigid body is to rotate it without moving it as a whole.
- The movement of the whole body is not possible because the resultant force is zero in the case of forces forming a couple.
- The perpendicular distance between the lines of action of two forces forming the couple is called the *arm* of *couple*.
- Thus, in Figure, two equal forces of magnitude *P* and acting at points *A* and *B* in the opposite direction form a couple with *AB* as arm of the couple.



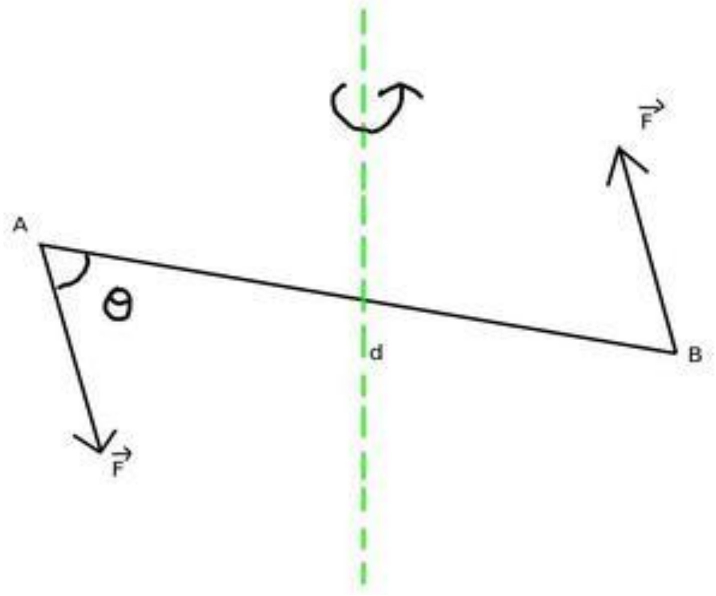
- *The moment of a couple is known as torque which is equal to one of the forces forming the couple multiplied by arm of the couple.*

COUPLE

- *When two equal and opposite forces are applied simultaneously at different points on a body, their resultant force is zero, but both these equal and opposite forces try to rotate the body in the same direction. These forces are called Couple.* Such a force pair has a tendency to rotate that body.
- Consider A and B as the two points on a body on which two forces F of the equal result are applied in opposite directions parallel to each other. Due to the effect of this force, this body starts rotating in the anti-clockwise direction or tries to rotate about the axis passing through 0.
- Keep in mind that if the line of action of these two equal forces is the same or they act at the same point, then they will cancel each other.
- A torque operates on a body as a result of a couple's activity, causing the body to spin around a fixed point.

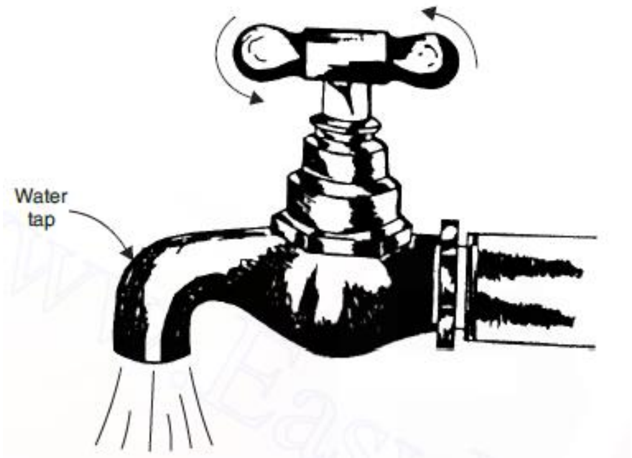
$$\tau = F \times r$$ (which denotes the torque applied on the body)

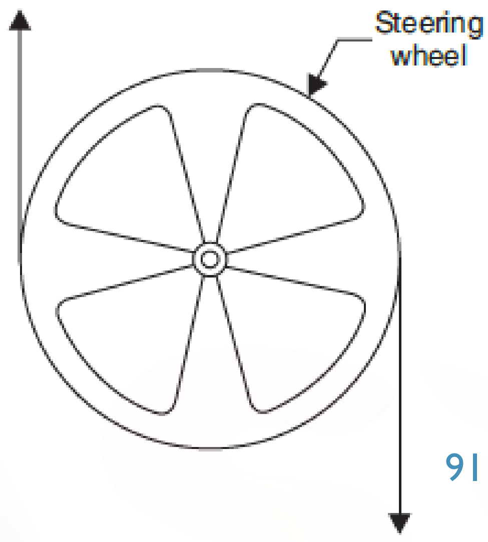
- The magnitude of torque equals the quantity of force multiplied by the perpendicular distance between the two forces.



EXAMPLES: COUPLE

1. Opening or closing the spout of the tap.
2. Turning the cap of a pen.
3. Winding a watch or clock with a key.
4. When steering a moving bicycle, we apply force to the handle with our hands.
5. While opening and closing the lid of the device, a pair of forces is applied with the fingers.
6. Keeping the pencil between the palms of both hands, the pencil starts rotating when the palms are run in the opposite direction.
7. While holding the ends of the rope with the hand while running the hand in the opposite direction, the churn starts rotating, this is also an example of a couple.



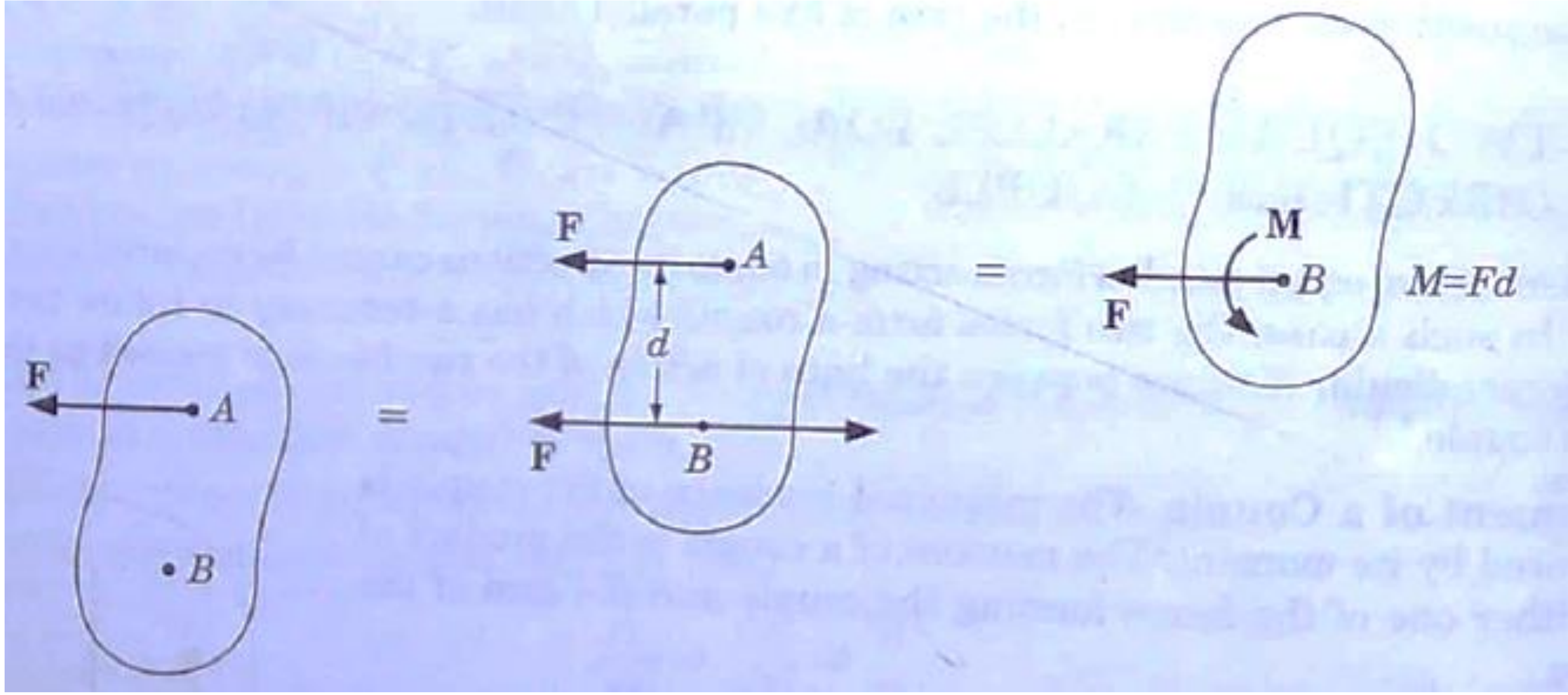


CHARACTERISTICS OF THE COUPLES

- As the two forces that make up the couple are equal and opposing, the couple does not create translational motion.
- When it operates on a body, the net resulting force on the body is zero.
- Since, the algebraic sum of the moments of the two forces around any point in their plane is not zero, it causes pure rotational motion in the body.
- A couple’s moment about any point in its plane is constant in size and direction.

RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE

- Consider a force F acting on a body at the point A.
- This is to be replaced by a force acting at some point B together with a couple as shown.
- Introduce two equal and opposite forces at B, each of magnitude F and acting parallel to the force at A.



RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE

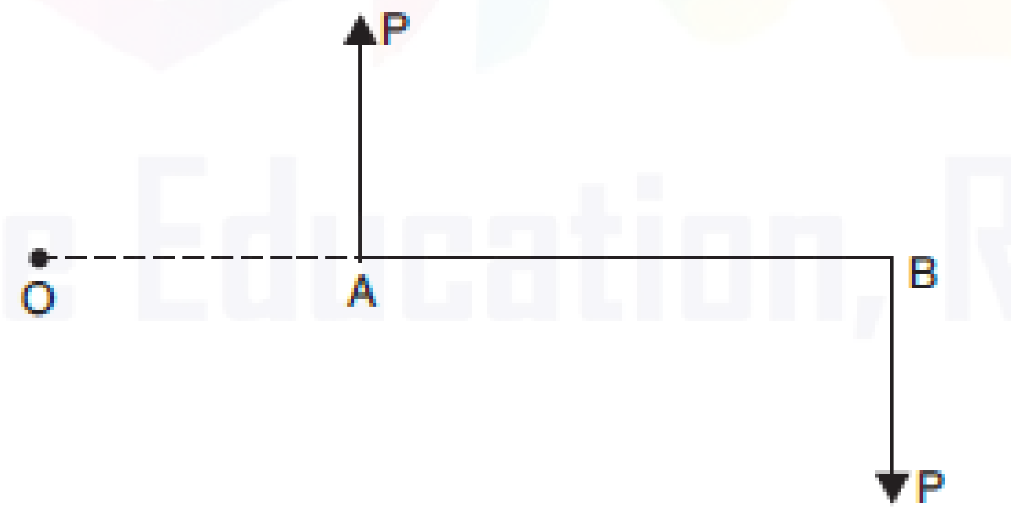
- From the principle of superposition, the second system of forces is equivalent to the single force acting at A.
- Of the three equal forces, consider the two forces acting in opposite directions at point A and B.
- They form a couple of moment.

$$M = F \times d$$

- Thus, the original force F acting at point A can be replaced by a force F applied at another point B, together with a couple of magnitude F x d.
- The distance d being the perpendicular distance between the lines of action of the forces at A and B.

PROPERTIES OF A COUPLE

- **1. The algebraic sum of the moments of the forces forming a couple about any point in their plane is constant.**
- Let two parallel and unlike forces be of magnitude *P* each forming a couple *P* × *AB* where points *A* and *B* are the points where forces *P* and *P* act. (Refer figure a)



(a)

PROPERTIES OF A COUPLE

- Moments about $O = P \times OB - P \times OA$

  $= P(OB - OA) \quad = P \times AB$ [Refer Figure (*a*)]

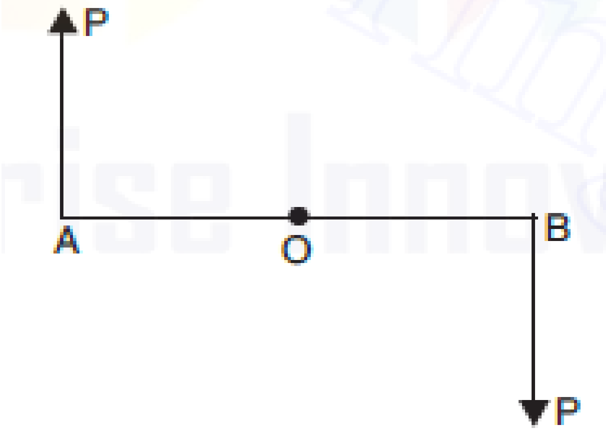
- Moments about $O = P \times OB + P \times OA$

  $= P(OB + OA) \quad = P \times AB$ [Refer Figure (*b*)]

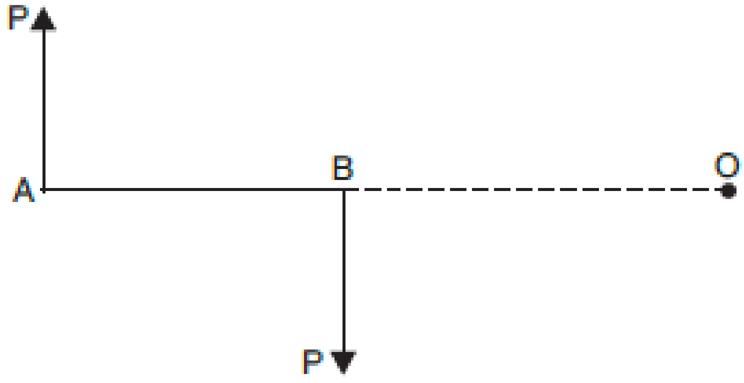
- Moments about $O = P \times OA - P \times OB$

  $= P(OA - OB) \quad = P \times AB.$ [Refer Figure (*c*)]

- In all the three cases, we find that the sum of the moments in each case is independent of the position of the point *O*, and depends only on the constant arm of the couple, so **the algebraic sum of moments of the forces forming a couple about any point in their plane is constant.**



(b)



(c)

PROPERTIES OF A COUPLE

2. Any two couples of equal moments and sense, in the same plane are equivalent in their effect. This result is quite useful as it clearly states that moment is the only important thing about a couple. Thus, in a couple we may change the magnitude or direction of the forces or the arm of the couple itself without changing its effect provided that the new couple with changed values has the same moment in the same sense.

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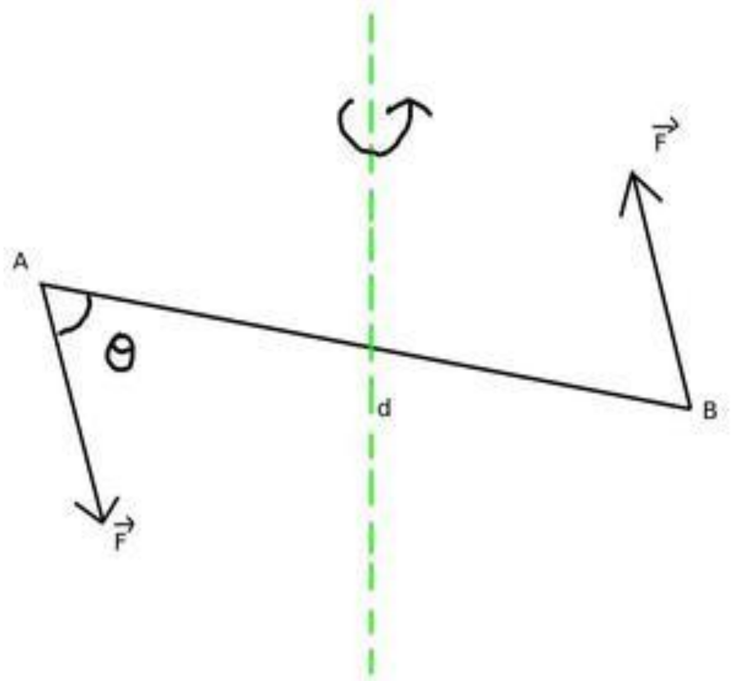
4. **A force acting on a rigid body can be replaced by an equal like force acting at any other point and a couple whose moment equals the moment of the force about the point where the equal like force is acting.**

5. **Any number of coplanar couples are equivalent to a single couple whose moment is equal to the algebraic sum of the moments of the individual couples.**

MOMENT OF THE COUPLE

- *The tendency of a force is to rotate a body. It is measured by the moment of the force. The product of one of the two forces of a Couple and the perpendicular distance between their lines of action (called the arm of the Couple) is called the* ***Moment of Couple****.*
- Mathematically, moment of couple is defined as the product of the force and the perpendicular distance between the lines of action of the two forces.
- Here, the perpendicular distance between the lines of action of two forces is also called the **arm of the Couple**. That is the moment of force is also equal to the product of the applied force and the arm of a pair of forces.
- Thus, the moment of pair of forces is given by, $\tau = F \times d$ …….(1)
- It is clear from the formula for the moment of a pair of forces that the moment of a pair of forces will be greater if The magnitude of the force is greater, and The arm of the pair of forces is longer, i.e. the perpendicular distance between the lines of action of the two forces is greater.
- The SI unit of Moment of Couple is Newton-Meter (N m). And the dimensional formula of Moment of Couple is $[ML^2T^{-2}]$.

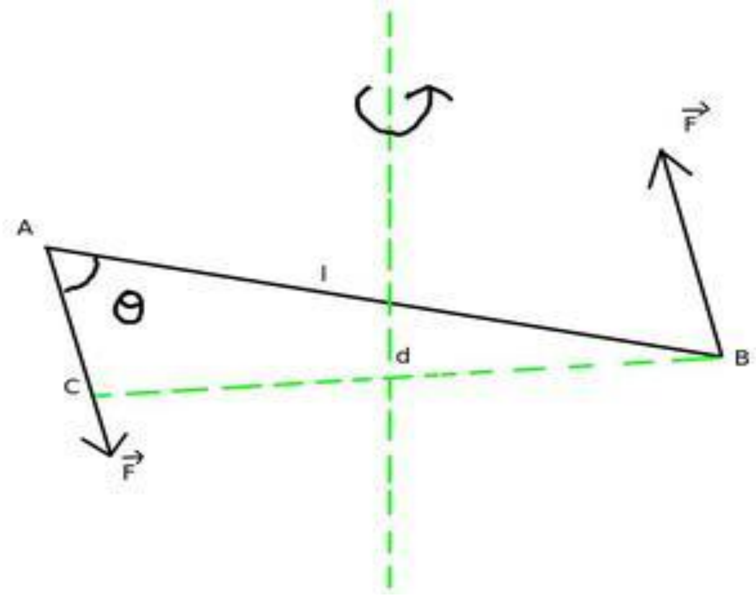


TYPES OF MOMENT OF COUPLE

- **Positive moment:** When the body tends to rotate in the anticlockwise direction under the influence of a Couple, then its moment is called positive moment
- **Negative moment:** When the force pair tends to rotate in the clockwise direction, then its moment is called negative moment.

VECTOR FORM OF MOMENT OF COUPLE

- *Lets draw BC perpendicular to the line of action of the force $\vec{F}$ acting from point B to A. Therefore,* $\angle BAC = \theta$.
- *And,* $\boldsymbol{\tau = F \times BC}$
- *But from right angle ΔABC:* $\sin\theta = BC / AB$, *or* $\boldsymbol{BC = AB \sin\theta}$
- *Now, putting values in equation (1),*
- $\boldsymbol{\tau = F \times AB \sin\theta}$ or $\boldsymbol{\tau = F \times l \sin\theta}$ …….(2)
- $\boldsymbol{= l \times F \sin\theta}$
- $\boldsymbol{\vec{\tau} = \vec{l} \times \vec{F}}$
- *This is the expression of Moment of Couple in vector form, where* $\vec{l} = \vec{AB}$



ENGINEERING APPLICATIONS OF MOMENTS

- Some of the important engineering applications of moments are :

1. The levers (simple curved, bent or cranked and compound levers).
2. The balance.
3. The common steel yard.
4. Lever safety valve.

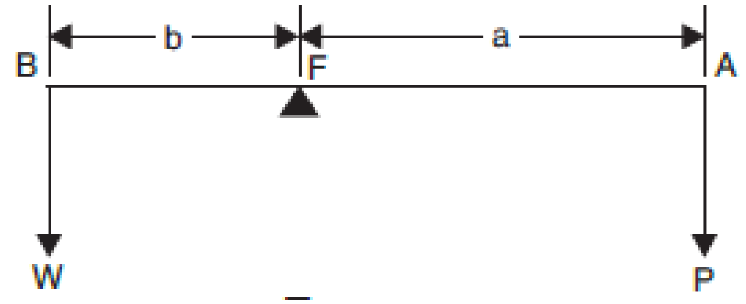
ENGINEERING APPLICATIONS OF MOMENTS

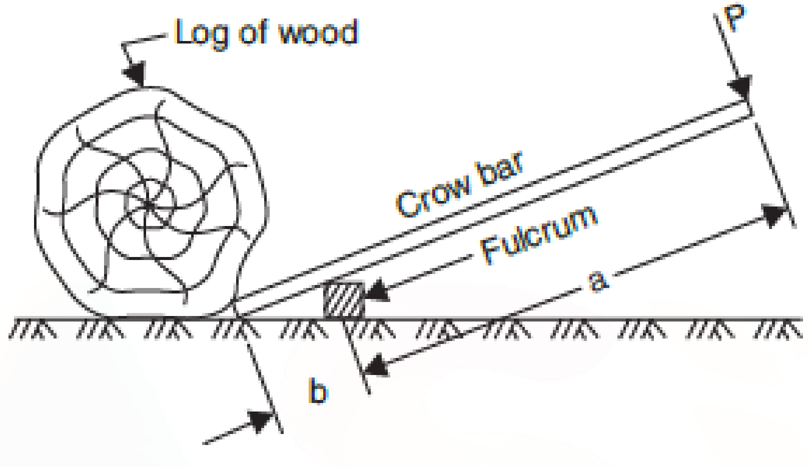
The Lever. The lever is defined as a *rigid bar, straight or curved which can turn about a fixed point* called the **‘fulcrum’.**

- It works on the *principle of moments i.e.*, when the lever is in equilibrium the algebraic sum of the moments, about the fulcrum, of the forces acting on it is zero.
- The principle of lever was first developed by Archimedes.
- Some common examples of the use of lever are :
- (*i*) Crow bar ;
- (*ii*) A pair of scissors ;
- (*iii*) Fire tongs etc.

ENGINEERING APPLICATIONS OF MOMENTS: THE LEVER.

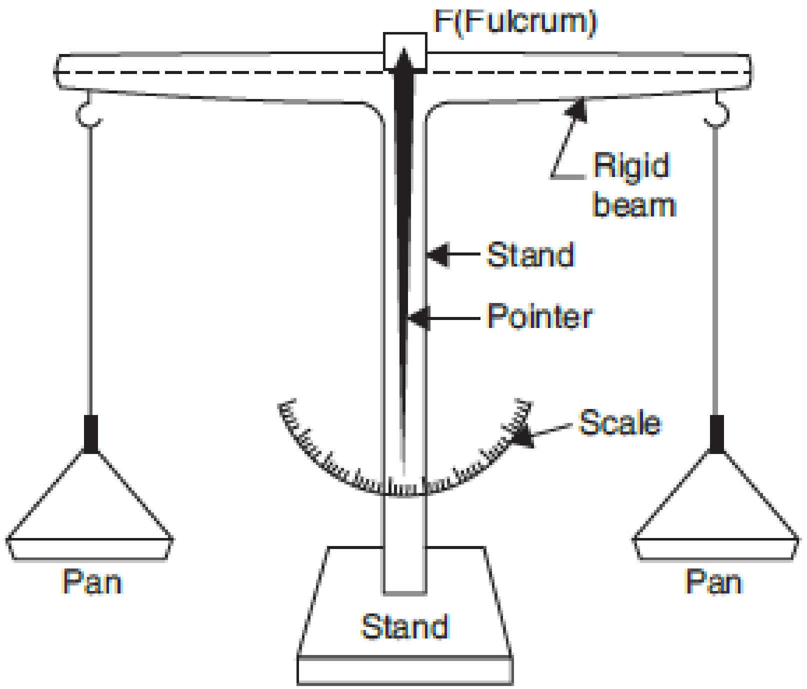
- **Power arm.** The perpendicular distance between the fulcrum and the line of action of forces is known as *power arm*.
- **Weight arm.** The perpendicular distance between the fulcrum and the point where the load/weight acts is called *weight* (or load) arm.
- The principle of moments is applicable when the lever is in equilibrium.
- Taking moments about *F* $P \times a = W \times b$
- Mechanical advantage of the lever $= \frac{W}{P} = \frac{a}{b} = \frac{\text{power arm}}{\text{weight arm}}$
- or **Power × power arm = weight × weight arm.** This is known as the *principle of the lever.*
- Figure 2 shows a crow bar used to move a heavy log of wood with the help of a small effort applied at its end with fulcrum suitably placed





ENGINEERING APPLICATIONS OF MOMENTS: THE BALANCE

- Figure shows a common balance which is just a lever of the first type, a device employed for weighing goods.
- It consists of a rigid beam having two scale pans suspended from each end.
- The beam can turn freely about a fulcrum *F* which is outside the beam but rigidly connected with it.
- A pointer is attached to the beam at its middle point *M* and a scale is also provided.
- For true balance the following conditions must be satisfied :

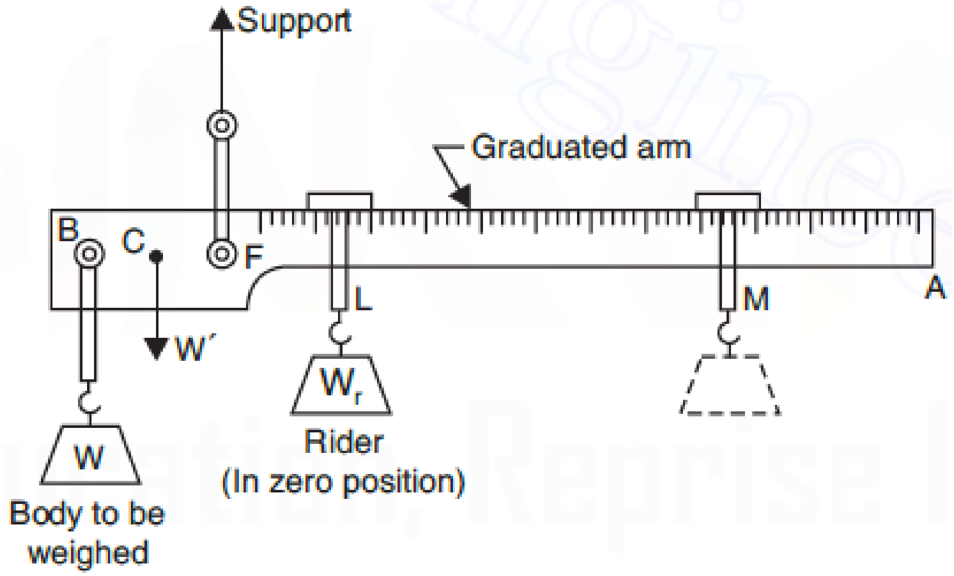


1. The arms of the balance must be of equal size.
2. The weights of scale pans must be equal.
3. The fulcrum, the centre of rigid beam and *c.g.* of the beam including its connected parts must lie on the line perpendicular to the beam.

- If the above conditions are not satisfied the problem of finding the unknown values from known values may be solved by taking moments about the fulcrum *F*.

ENGINEERING APPLICATIONS OF MOMENTS: THE COMMON STEEL YARD

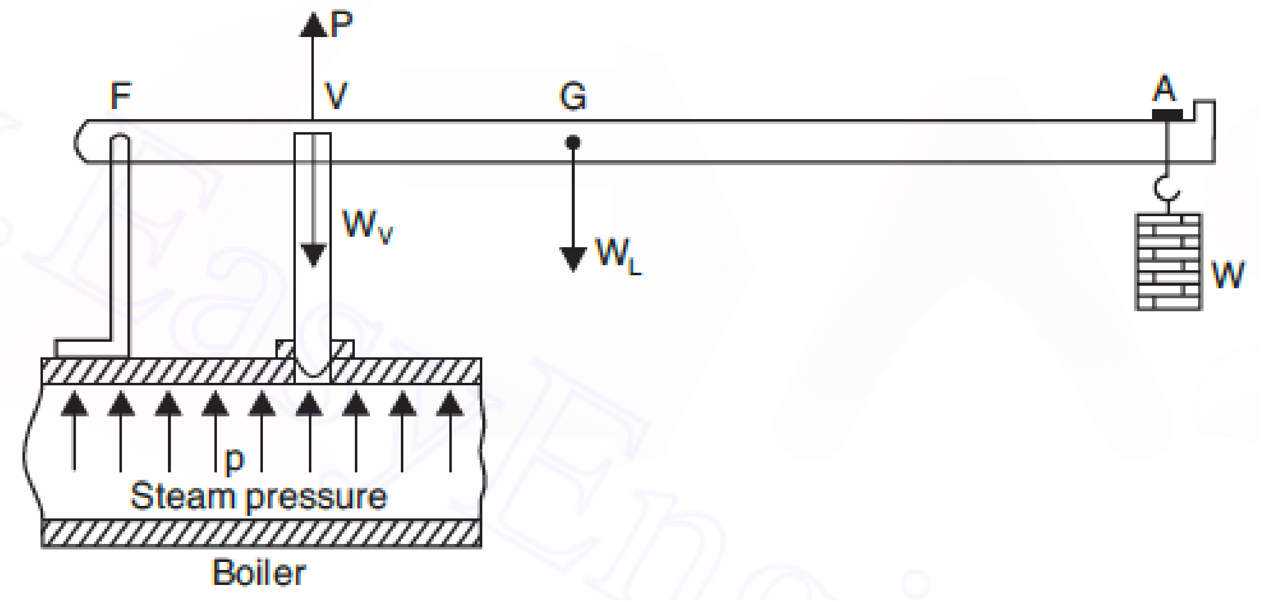
- The common steel yard is a device used for weighing goods based on type I lever.
- It consists of a rod *AB* which can move about a fixed fulcrum *F* which is kept near one of the ends.
- The rod is made heavier on the shorter side so that the *c.g.* of the whole rod and pan which is attached to the end of the shorter side lies on the shorter side.
- The rider (movable weight) is kept on the longer side as shown in Figure.



- The longer arm on which the rider moves is graduated.
- The position of rider determines the weight of the body acting on shorter side end when the rod rests in a horizontal position.
- It may be noted that the steel yard will give the value of the weight of the body in the same units in which its calibration is made.

ENGINEERING APPLICATIONS OF MOMENTS: LEVER SAFETY VALVE

- A lever safety valve is *a boiler mounting the purpose of which is to keep the steam pressure in the boiler upto certain safe values* and to the release the same when the pressure increases the safe limits.
- Figure consists of a valve *V* rigidly connected with the lever *FA*, whose fulcrum is at *F*.
- At the end *A*, a weight *W* is hung which exerts moment on to the valve to keep it placed on its seat against the steam pressure from below, which exerts a counter moment about the fulcrum *F*.
- As soon as the moment due to steam pressure increases, the valve lifts up its seat and releases excessive pressure into the atmosphere.
- When the steam pressure inside the boiler falls down to safe value the valve automatically occupies its seat and stops further escape of steam



RESULTANT OF A COPLANAR, NON-CONCURRENT NON-PARALLEL FORCE SYSTEM

(*i*) The magnitude, direction and position of the resultant of a given coplanar, non-concurrent, non-parallel force system are found analytically as follows :

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

where, ΣH = algebraic sum of the horizontal components of all the forces.

ΣV = algebraic sum of vertical components of all the forces.

(*ii*) The *direction* of the resultant is determined by using the relation,

$$tan\alpha = \frac{\Sigma \text{V}}{\Sigma \text{H}}$$

(*iii*) The *position* of the resultant is determined by taking moments of all the rectangular components of forces about a point in their plane and equating the algebraic sum of moments of all the forces to that of the resultant by using the relation,

Moment of resultant '*R*' about point = algebraic sum of rectangular components of all forces.